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BASIC

THERMODYNAMICS

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THERMODYNAMIC

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* [BASIC CONCEPTS OF THERMODYNAMICS] *

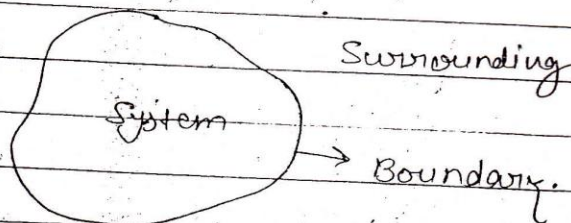
Thermodynamics is the science of energy transfer and its effects on properties of system.
* The main aim of T.D. study is to convert disorganized form of energy (heat) into organized form of energy (work) in an efficient manner.

[SYSTEM] It is a region in space upon which the study is focused or concentrated.

[SURROUNDING] Any thing external to the system is known as surrounding.

Boundary → The separation b/w system & surroundings is known as boundary.

[NOTE] → Boundary can be rigid or flexible, it can be real or imaginary.



[NOTE] → Universe is equal to system plus surroundings.

{ Universe = system + surrounding }

* TYPES OF SYSTEM *

Types of system	mass tran.	Energy trans.	Example
closed	X	✓	Piston-cylinder w/o valves
open	✓	✓	turbine, Compressor, pump, boiler, Condenser
isolated	X	X	Universe, Hot coffee in well insulated flask.
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: Microscopic & Macroscopic approach of thermodynamics →:

* In microscopic approach the behaviour of individual molecules is taken into consideration.

This approach is also known as statistical thermodynamics. This approach is useful at higher altitudes (lower densities).

* In macroscopic approach average behaviour of molecules is taken into consideration. This approach is also known as "classical thermodynamics".

* THERMODYNAMIC EQUILIBRIUM *

A system is said to be in thermodynamic equilibrium if it is in

- (1) Thermal equilibrium [Equality of temperature]
- (2) mechanical eq. [Equality of forces/pressure]
- (3) chemical " [Eq. of chemical potential]

* PURE SUBSTANCE → A substance is said to be a pure substance if it is

- (a) Homogeneous in chemical composition
- (b) homogeneous in chemical aggregation (bonding).

Steam H_2O	$H_2 + O$	$H_2 + O_2$
water H_2O	water (H_2O)	water.

(a) ✓

(b) ✓ Pure Sub.

Eduwite

(a) ✓

(b) X
Not pure Sub.

(a) X

(b) X
Not pure Sub

simba

Remark $\rightarrow$ Phase has nothing to do with the concept ^② of a pure substance.

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* [PROPERTY OF A SYSTEM] *

Any measurable characteristic is a property.

Properties are of two types.

imp

1 INTENSIVE PROPERTIES (Independent of size or mass)

imp

eg - Pressure, Temp., density, viscosity, thermal conductivity etc.

2

EXTENSIVE PR. (Depend on mass or size).

eg $\rightarrow$ Volume, mass and all form of energy (kinetic, potential etc.).

m		$m/2$	$m/2$	} Extensive prop.
v		$v/2$	$v/2$	
p	$\Rightarrow$	p	p	} Intensive prop.
T		T	T	

NOTE $\Rightarrow$ ① Ratio of two extensive properties is an 'intensive property'.

② Specific properties are intensive properties. Examples - specific volume (v), specific internal energy (u), specific enthalpy (h), specific entropy (s).

$\hookrightarrow$ Important points w.r.t properties

① Properties are point functions / state functions.

② Properties are independent of past history.

③ Properties are exact differentials.

PROCESS :- A change of state is known as a process.

→ State of a system :- Condition of a system is known as state of a system. If properties are fixed, state is fixed and hence condition is also fixed.

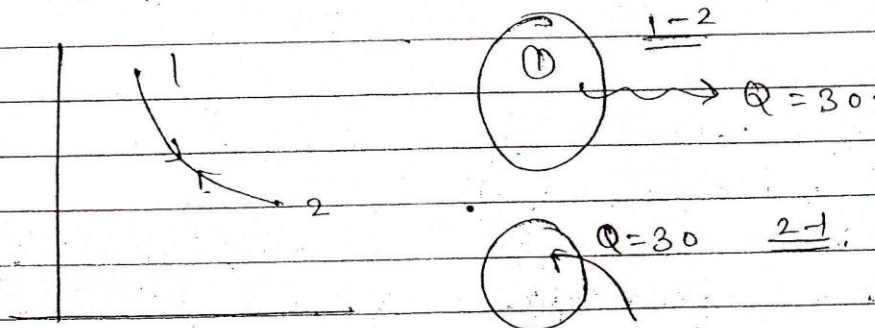
→ REVERSIBLE & IRREVERSIBLE PROCESSES

* A process is said to be a reversible process if when reversed in direction follows the same path without leaving any effect on system & surrounding.

* Reversible process is the most efficient process.

* A process which is not a reversible process is an irreversible process. All practical processes are irreversible processes.

* Friction is one of the main reasons for making a process irreversible.



System

Surrounding

$$Q_{1-2} = -30 \text{ unit}$$

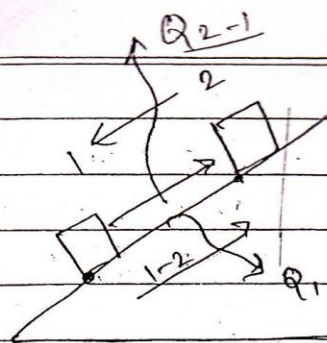
$$Q_{1-2} = +30$$

$$Q_{2-1} = 30 \text{ unit}$$

$$Q_{2-1} = -30$$

Eduwrte

No effect on system & surr
→ Reversible



(3)

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Q_{1-2} = [energy (heat) is produced due to friction & taken by Surrou.]

here $\underline{1-2}$ Q_{1-2} is given to surrounding

$2-1$ Q_{2-1} is given to surr.

In both the cases, surrounding gains energy
 $\Rightarrow$ Not reversible.

* QUASI-STATIC PROCESS :-

$\downarrow$
almost

$\downarrow$
Rest

$\rightarrow$ A process which is carried out in a slow manner with small gradient is known as a quasi-static process. Frictionless quasi-static process is a reversible quasi-static process.

* <Thermodynamic cycle> : (Cyclic process) $\rightarrow$:- A system is said to have undergone a cycle if the initial and final points are same. The change in properties for a cycle is equal to zero. Minimum number of processes required for a cycle are two.

* <Gibbs phase rule> According to Gibbs phase rule

$$P + F = C + 2$$

P : No. of phases

F : minimum no. of independent intensive variables required to fix the state.

C : No. of components (chemical component constituents)

Eduwrite

ex →

①		Here	
	O_2 gas	$P=1$ (gas)	$\Rightarrow P+F=C+2$
		$C=1$ (O_2)	$1+F=1+2$
			$[F=2]$

eg ②

	Steam	$P=2$	$\Rightarrow 2+F=2+1$
	Water	$C=1$ (H_2O)	$[F=1]$

eg ③

	ice	$P=3$	$\Rightarrow 3+F=2+1$
	Steam	$C=1$	$[F=0]$
	water		

The degree of freedom at tripple point is zero.

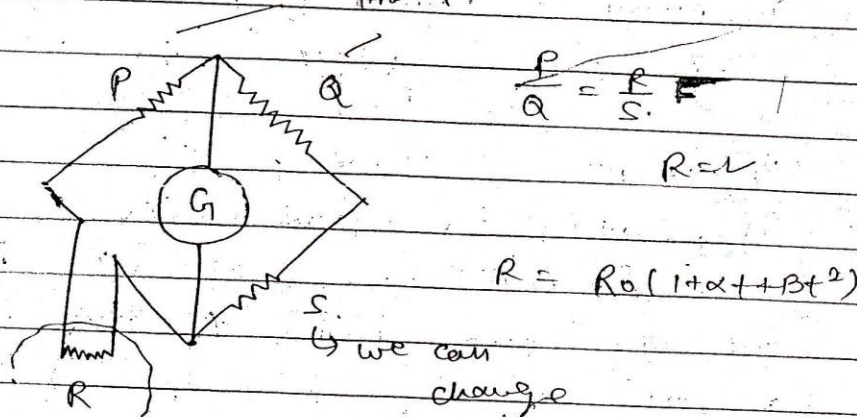
* ZEROETH LAW OF T.D. (CONCEPT OF TEMPERATURE)

Statement → when A is in thermal eq. with B and B is in thermal eq. with C separately, then A & C are in thermal equilibrium.
In zeroth law of T.D. one body acts as a thermometer.

* [Thermometric Principle] In measuring temp. the parameter which varies with temp. is found first, with the help of this parameter the temperature is found. The property which helps in finding temp. is known as thermometric property.

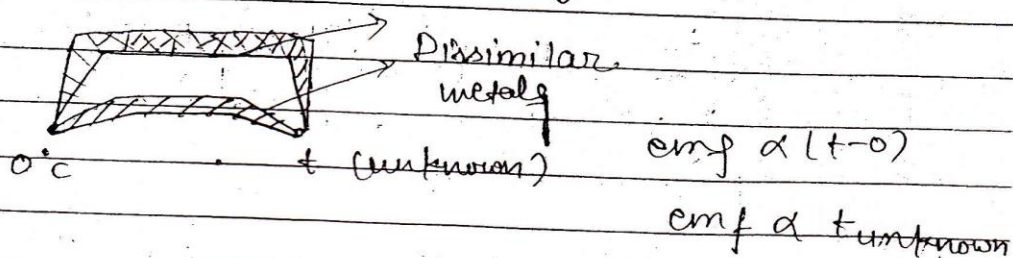
* [Types of thermometers] *

- 1- Resistance thermometer is based on wheatstone bridge principle and in this thermometer resistance is thermometric property. Also known as Thermistor.



exposed to
unknown temp.

- 2- Thermocouple → Thermocouples are based on Seebeck-effect. Acc. to Seebeck-effect when two dissimilar metals are joined to form two junctions and if these two junctions are maintained at different temperatures, emf or voltage is generated and this emf is proportional to temp. difference b/w two junctions. In thermocouples emf plays a role of thermometric property.



<Constant volume gas thermometer> In this thermometer pressure plays the role of thermometric property.

<Constant pressure gas thermometer> In this thermometer volume plays the role of thermometric property.

[TEMPERATURE SCALES]

Temperature scales are arbitrary. In centigrade scale ice point (freezing point) is taken as 0°C and steam point (boiling point) is taken as 100°C .

$$K = ^{\circ}\text{C} + 273.15$$

* {Method used before 1954 for measuring temp}

This method is based on two reference temperatures (ice point is 0°C & steam point is 100°C).

$$t = ap + b$$

t : unknown temp. to be found

a & $b \rightarrow$ constants.

$p \rightarrow$ any thermodynamic property

$$t_i = ap_i + b \quad (\text{ice point})$$

$$t_s = ap_s + b \quad (\text{steam point})$$

$$0 = ap_i + b \quad \text{--- ①}$$

$$100 = ap_s + b \quad \text{--- ②}$$

from ① and ② $\left\{ a = \frac{100}{p_s - p_i} \right\}$

hence $b = -a p_i = \frac{-100}{p_s - p_i} \cdot p_i$ 209 (5)

$$t = a p + b$$

$$t = \frac{100}{p_s - p_i} \cdot p - \frac{100 p_i}{p_s - p_i}$$

$$t = \frac{100}{(p_s - p_i)} (p - p_i)$$

if the property 'P' is known then the corresponding temp. t can be found

* [method used after 1954 for measuring temp.]

⇒ Its based on single fixed temperature i.e. tripple point of water.

* Tripple point of water is taken as 0.01°C (273.16 K).

* for temp. measurement suppose we have constant volume thermometer.

$$\Rightarrow P \propto T$$

$$P = cT$$

at tripple point $\cdot P_{tp} = c T_{tp} \Rightarrow \left\{ c = \frac{P_{tp}}{T_{tp}} \right\}$

so $P = \frac{P_{tp}}{T_{tp}} \times T$

OR $T = \frac{P}{P_{tp}} \times T_{tp}$

$$T = 273.16 \left[\frac{P}{P_{tp}} \right] \rightarrow \text{if } P \text{ is known}$$

we can find T.

↓

in Kelvin

OTE → Ideal gas thermometer is independent of material of construction.

The readings t_A & t_B of two thermometers agree at ice point and steam point and are related by the equation $t_A = l + mt_B + nt_B^2$ b/w these temperatures. l, m & $n \rightarrow$ constants. when

both are immersed in oil A reads 51°C & B reads 50°C .

Determine the reading of A when B reads 25°C .

Discuss the question which thermometer is correct.

tion)

we have

$$t_A = l + mt_B + nt_B^2 \quad \text{--- (A)}$$

Given at ice point $t_A = t_B$

$$\text{ie } t_A = t_B = 0^\circ\text{C}$$

from (A)

$$0 = l + 0 + 0$$

$$\boxed{l = 0}$$

$$\boxed{t_A = mt_B + nt_B^2}$$

(B)

Given, at steam point $\Rightarrow t_A = t_B$

$$\text{ie } t_A = t_B = 100^\circ\text{C}$$

from (A)

$$100 = 0 + m \times 100 + n(100)^2$$

$$1 = m + 100n \quad \text{--- (1)}$$

it is given (3rd condition)

$$t_A = 51 \text{ \& } t_B = 50^\circ\text{C}$$

from (B)

$$51 = 50m + n(50)^2$$

$$51 = 50m + 2500n \quad \text{--- (2)}$$

from (1) and (2) $m = 1.04, n = -4 \times 10^{-4}$.

and if $t_B = 25^\circ\text{C}$ then $t_A = ?$

from (B)

$$t_A = 1.04 t_B + 4 \times 10^{-4} t_B^2$$

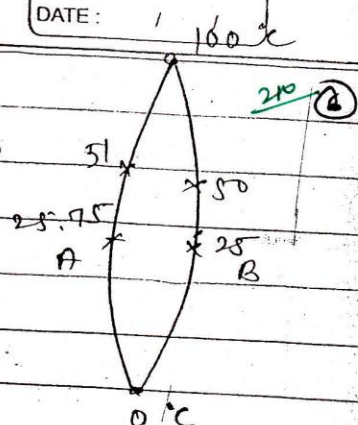
$$\text{Put } t_B = 25^\circ\text{C}$$

we

get

$$\boxed{t_A = 25.75^\circ\text{C}} \quad \text{A}$$

Though the end point temp. are same, it doesn't mean intermediate temp. are also same.



Q2) Assertion - If Hg. and alcohol thermometers read exactly at ice point and steam points, then these two thermometers will give exactly the same reading at 50°C.

Reason - Temp. scales are arbitrary.

Ans → Reason Assertion is wrong & reason is right.

Question 3) LIST I

- A. Mercury in glass
- B. Thermocouple
- C. Thermistor
- D. Const. volume thermometer

LIST II

- 1 - Pressure
- 2 - Resistance
- 3 - Volume
- 4 - e.m.f.

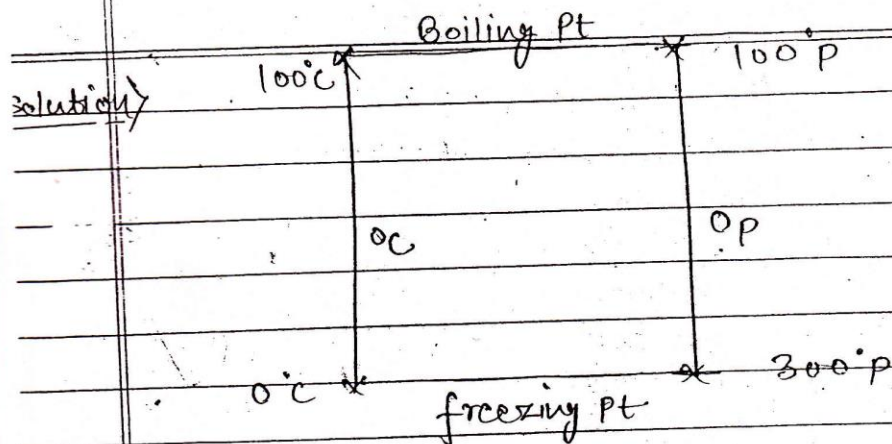
Ans → A-3, B-4, C-2, D-1.

Question) which of the following are intensive properties
1 - K.E., 2 - Thermal conductivity
3 - Pressure 4 - Volume.

(a) 1 & 2 (b) 2, 3 & 4 (c) 2 & 3 (d) 1, 3 & 4
only K.E. & P are extensive properties.

Question) In a new temp. scale ($^{\circ}P$) the boiling point of water is $100^{\circ}P$ and the freezing point is $300^{\circ}P$. Find the reading corresponding to $0^{\circ}P$ on Celsius scale.

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IMPRemark →

whenever there is the relationship b/w the two tempo scales is not given, always consider a linear variation b/w them.

let $^{\circ}P = ax^{\circ}C + b$

at freezing point $300 = ax + b$

$b = 300$

at boiling point $100 = ax + 300$

$a = -2$

hence $^{\circ}P = -2x^{\circ}C + 300$

if $^{\circ}P = 0^{\circ}P$

$0 = -2C + 300$

$C = 150^{\circ}C$ Ans.

VERY
IMPORTANT

CHAPTER-2

ENERGY INTERACTIONS

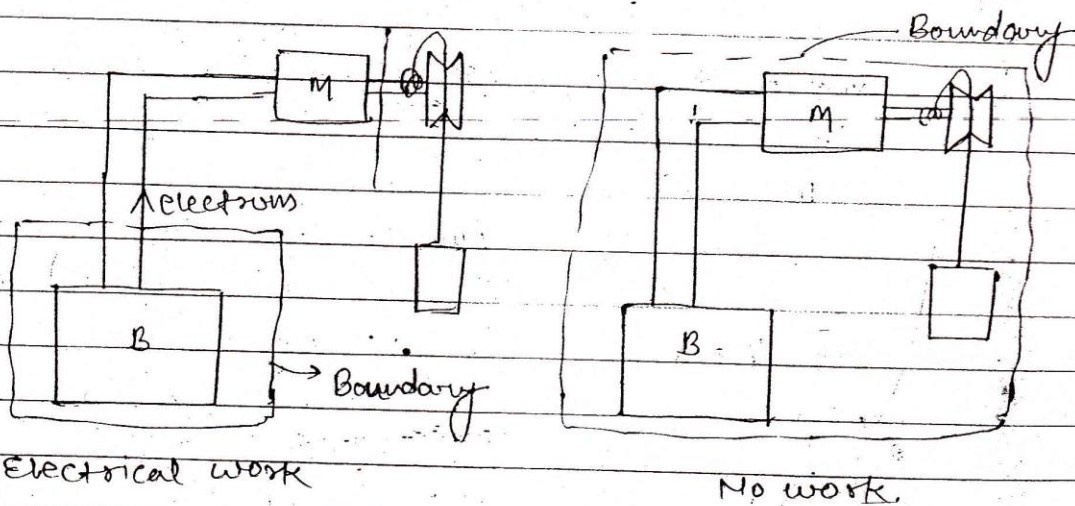
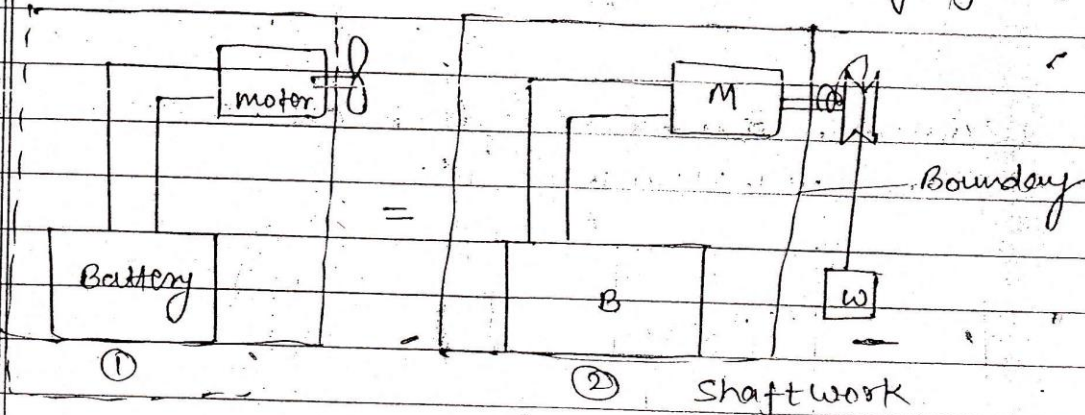
[WORK AND HEAT]

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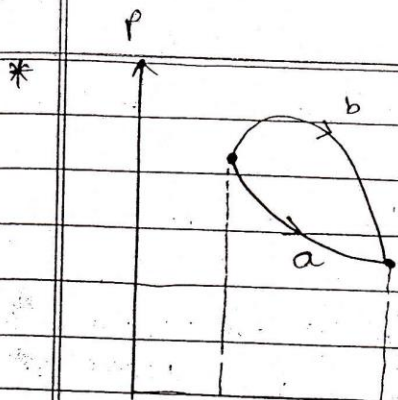
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THERMODYNAMIC WORK → Work is said to be done by the system if the sole effect on things external to the system can be equated to raising of weights. weights may not be actually raised but the effect can be equated to raising of weights.



Imp

NOTE → Work transfer occurs only when it crosses the boundary and hence work transfer is a boundary phenomenon.



from fig.

w for path a $\neq w$ for path b.

$$w_a \neq w_b$$

(even though end points are same).

NOTE → Though the end points are same for the paths a & b, work-transfer is not same for both paths. Because areas are different for different paths and therefore work transfer is a path function and it is inexact differential (δw & δw).

Imp
*

Conditions for applying the equation $w = \int P dv$.

(1) System must be a closed system.

(2) Work should cross the boundary.

3 → Process must be a reversible process.

NOTE → Work transfer occurs ^{only} when there is a process, taking place. Therefore work transfer is associated with the process. If there is no process then there is no work transfer. Work is never possessed by system.

** Work transfer in various closed system, or non-flow processes → :-

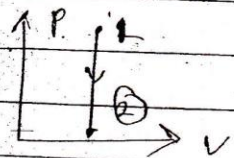
(1) CONSTANT VOLUME (or) ISOCHORIC (or) ISOMETRIC PROCESS →

Volume = Constant

$$dv = 0$$

$$w = \int P dv \Rightarrow \boxed{w = 0}$$

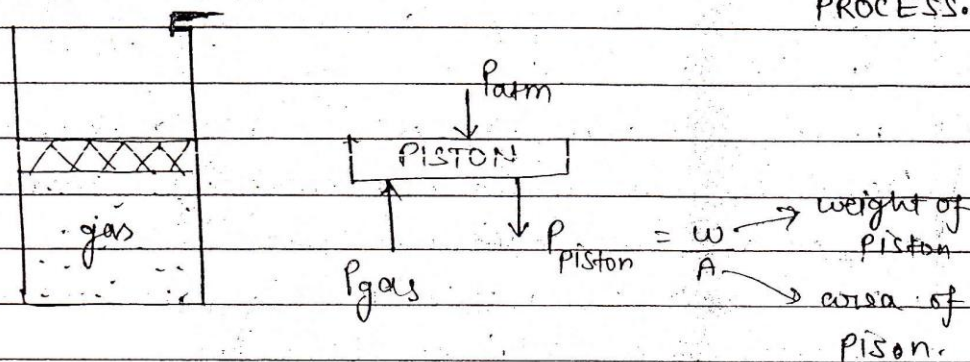
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whenever there is a rigid container $\rightarrow$ constant volume.

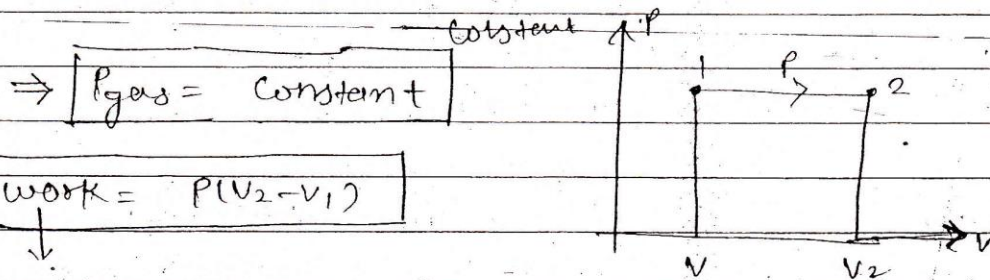
* Constant volume closed system work $= 0$.

2) CONSTANT PRESSURE (or) ISOBARIC (or) ISOPHASTIC PROCESS.



ie $P_{gas} = P_{atm} + P_{piston}$.

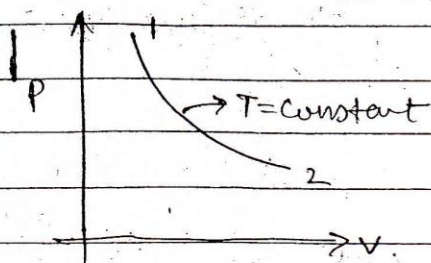
$$P_{gas} = (P_a) + \frac{w}{A} \quad \text{Constant}$$



"Reversible closed system, constant pressure work."

3) CONSTANT TEMP (or) ISOTHERMAL PROCESS $\rightarrow$

$P \cdot V = \text{Constant} \rightarrow$ (Rectangular Hyperbola)



$$\{P \cdot V = C = nRT\}$$

$$w = \int P \, dv$$

$$w = \int_{V_1}^{V_2} \frac{C}{V} \, dv$$

$$w = c \ln \left(\frac{v_2}{v_1} \right).$$

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$$w = nRT \ln \left(\frac{v_2}{v_1} \right) = P_1 v_1 \ln \left(\frac{v_2}{v_1} \right).$$

closed system, reversible and constant temperature process work.

(4) [ADIABATIC PROCESS]

For an adiabatic process, heat transfer = 0 and $p v^\gamma = \text{constant}$.

γ : Adiabatic index and $\gamma \geq 1$.

we know

$$w = \int_{v_1}^{v_2} p dv.$$

$$w = \int_{v_1}^{v_2} \frac{c}{v^\gamma} dv = \int_{v_1}^{v_2} c \cdot v^{-\gamma} dv.$$

$$w = c \int_{v_1}^{v_2} v^{-\gamma} dv.$$

$$w = c \left[\frac{v^{-\gamma+1}}{-\gamma+1} \right]_{v_1}^{v_2}.$$

$$w = \frac{c}{-\gamma+1} \left[v_2^{-\gamma+1} - v_1^{-\gamma+1} \right]$$

$$= \frac{c}{-\gamma+1} \left[v_2^{-\gamma} \cdot v_2 - v_1^{-\gamma} \cdot v_1 \right]$$

$$= \frac{1}{-\gamma+1} \left[(c v_2^{-\gamma}) \cdot v_2 - (c v_1^{-\gamma}) \cdot v_1 \right]$$

$$= \frac{1}{-\gamma+1} \left[P_2 \cdot v_2 - P_1 v_1 \right]$$

$$= \frac{1}{-\gamma+1} \left[-P_1 v_1 - P_2 v_2 \right]$$

$$W = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1}$$

Adiabatic, closed system reversible work.

2) [POLYTROPIC PROCESS]

$Q \neq 0$

$$W = \frac{P_1 V_1 - P_2 V_2}{n - 1}$$

n : polytropic index.

- * A process is said to be a polytropic process if $PV^n = C$. where $n \rightarrow$ polytropic index.

Generally

$$1 < n < \gamma$$

Although it may be negative in some cases.

- * In polytropic process there is both heat and work transfer. But whereas in ~~an~~ adiabatic there is only work transfer.

- * All the processes discussed above can be considered

of the form $PV^K = C$

for isobaric process	$K = 0$
for isothermal "	$K = 1$
for polytropic "	$K = n$
for adiabatic "	$K = \gamma$
for ** constant volume "	$K = \infty$

I
N
C
R
E
A
S
I
N
G

value of K .

$\downarrow$ or

** for constant volume process

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$$PV^K = C$$

$$(PV^K)^{\frac{1}{K}} = (C)^{\frac{1}{K}}$$

$$(P)^{\frac{1}{K}} \cdot V = (C)^{\frac{1}{K}}$$

if. volume is to be constant
 $\Rightarrow K = \infty$

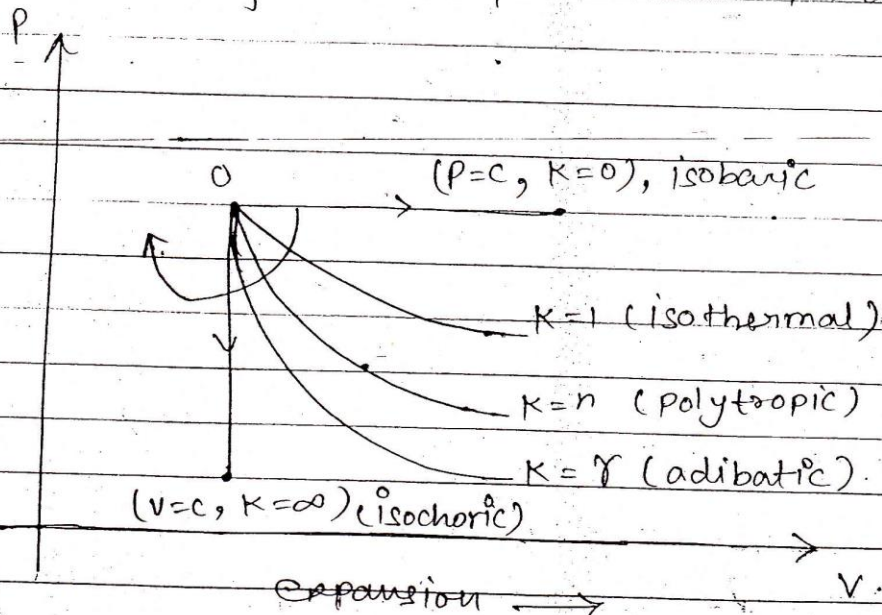
$$(P)^{\frac{1}{\infty}} \cdot V = \text{Constant}$$

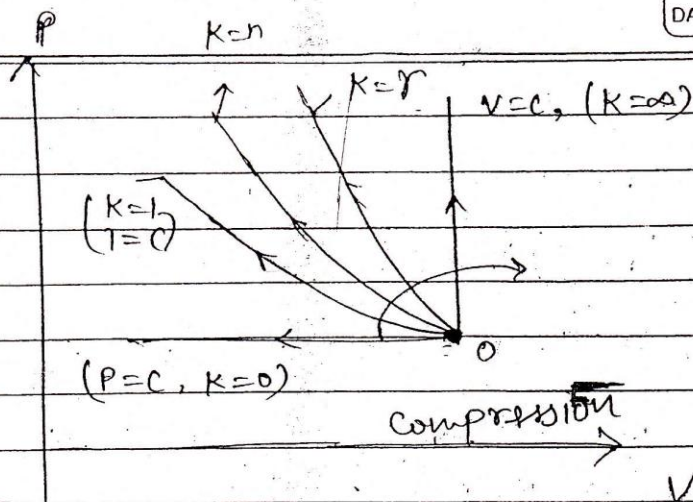
$$(P)^0 \cdot V = \text{constant}$$

$$\boxed{V = \text{Constant}} \text{ if } K = \infty$$

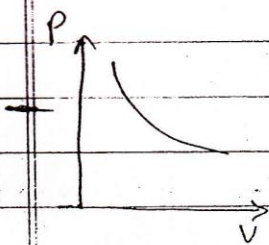
Ans

* [Representation of various processes on P-V diagram]





SLOPE OF ISOTHERMAL CURVE ON P-V DIAGRAM
FOR AN IDEAL GAS -



$$T = C$$

$$PV = mRT$$

$$PV = C$$

$$Pdv + vdp = 0$$

$$Pdv = -vdp$$

$$\boxed{\frac{dp}{dv} = -\frac{P}{V}}$$

Slope of isothermal curve - $-\frac{P}{V}$

[for adiabatic curve]

$$PV^{\gamma} = C$$

$$P[rV^{\gamma-1}dv] + v^{\gamma}dp = 0$$

$$P\gamma \cdot V^{\gamma-1} dv + v^{\gamma} dp = 0$$

$$P(\gamma) \frac{dv}{v} + dp = 0$$

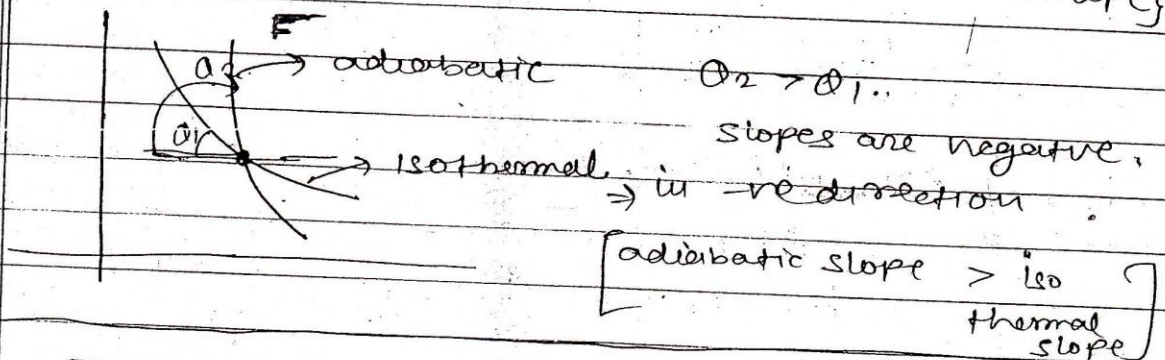
$$\frac{dp}{dv} = -\gamma \left[\frac{P}{V} \right]$$

$$\boxed{\frac{dp}{dv} = \gamma \left[-\frac{P}{V} \right]}$$

i.e. $\boxed{\text{Adiabatic slope} = \gamma (\text{Isothermal slope})}$ (11)

$$\boxed{\frac{\text{Adiabatic slope}}{\text{Isothermal slope}} = \gamma}$$

but $\gamma > 1 \Rightarrow \{\text{adiabatic slope} > \text{isothermal slope}\}$



IDEAL GAS EQUATION FOR VARIOUS PROCESSES -

(1) [CONSTANT VOLUME PROCESS]

$$V = \text{constant}$$

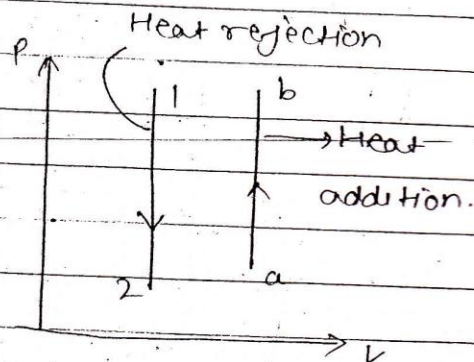
$$P V = m R T$$

$$P \propto T$$

$$\boxed{\frac{P_2}{P_1} = \frac{T_2}{T_1}}$$

as $P \uparrow \Rightarrow T \uparrow = \text{Heat addition}$

$P \downarrow \Rightarrow T \downarrow = \text{Heat rejection}$



(2)

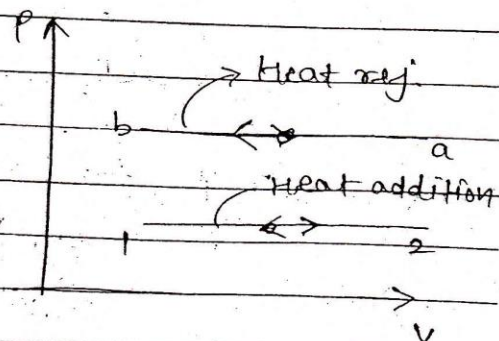
[CONSTANT PRESSURE PROCESS]

$$P = \text{constant}$$

$$\Rightarrow P V = m R T$$

$$\boxed{V \propto T}$$

$$\boxed{\frac{V_2}{V_1} = \frac{T_2}{T_1}}$$



[3] ISOTHERMAL PROCESS →

$$T = \text{constant}$$

$$PV = nRT$$

$$PV = \text{constant}$$

$$P_1 V_1 = P_2 V_2$$

$$\boxed{\frac{P_1}{P_2} = \frac{V_2}{V_1}}$$

[4] ADIABATIC PROCESS →

$$PV^\gamma = C \quad \text{--- (1)}$$

$$PV = nRT \Rightarrow P = \frac{nRT}{V}$$

$$\text{from (1)} \quad \frac{nRT}{V} V^\gamma = C$$

$$TV^{\gamma-1} = \frac{C}{nR} \Rightarrow \{TV^{\gamma-1} = \text{constant}\}$$

$$\boxed{T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}} \quad \text{--- (2)}$$

$$\text{we know } P_1 V_1^\gamma = P_2 V_2^\gamma \quad \text{--- (3)}$$

from (2) and (3)

$$\left(\frac{V_1}{V_2}\right) = \left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma-1}} \quad \& \quad \frac{V_1}{V_2} = \left(\frac{P_2}{P_1}\right)^{\frac{1}{\gamma}}$$

$$\Rightarrow \left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma-1}} = \left(\frac{P_2}{P_1}\right)^{\frac{1}{\gamma}}$$

$$\boxed{\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}}$$

hence for an adiabatic process (ideal gas)

$$\left[\begin{array}{l} P_1 V_1^\gamma = P_2 V_2^\gamma \\ T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \\ \frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \end{array} \right]$$

valid for ideal gas
undergoing adiabatic
process.

(5) POLYTROPIC PROCESS

$$\begin{aligned}
 P_1 V_1^n &= P_2 V_2^n \\
 T_1 V_1^{n-1} &= T_2 V_2^{n-1} \\
 \frac{T_2}{T_1} &= \left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}}
 \end{aligned}$$

Valid for ideal gas undergoing polytropic process.

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[CONVENTIONAL PROBLEMS]

Question (1) → A system undergoes three processes as shown in fig. 1-2 is isobaric process. 2-3 is adiabatic process with $\gamma = 1.4$ and 3-1 is isothermal process ($PV = C$). Find (i) V_2

(ii) Net work transfer.

SOL - Process 3-1 ($T = C$)

$$P_3 V_3 = P_1 V_1 \quad P_1 = P_2 = 400 \frac{\text{kN}}{\text{m}^2}$$

$$100 \times V_3 = 400 \times 1$$

$$V_3 = 4 \text{ m}^3$$

Process 2-3

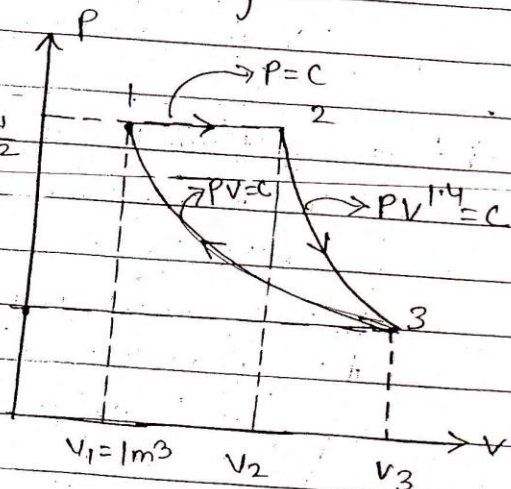
$$P_3 = 100 \frac{\text{kN}}{\text{m}^2}$$

$$P_2 V_2^{1.4} = P_3 V_3^{1.4}$$

$$400 \times V_2^{1.4} = 100 \times (4)^{1.4}$$

$$(V_2)^{1.4} = \frac{1}{4} \times (4)^{1.4}$$

$$V_2 = 1.486 \text{ m}^3 \quad \text{Ans}$$



$$(ii) W_{\text{net}} = \sum W = W_{12} + W_{23} + W_{31}$$

$$W_{12} = P_1 (V_2 - V_1) = 400 (1.486 - 1)$$

$$W_{12} = 194.4 \text{ kJ}$$

$$W_{2-3} = \frac{P_2 V_2 - P_3 V_3}{\gamma - 1}$$

$$= \frac{400 \times 1.486 - 100 \times 4}{1.4 - 1}$$

$$\{W_{2-3} = 4486 \text{ KJ}\}$$

$$W_{3-1} = P_3 V_3 \ln \left(\frac{V_1}{V_3} \right)$$

$$= 100 \times 4 \ln \left(\frac{1}{4} \right)$$

$$[W_{3-1} = -554.517 \text{ KJ}]$$

$$W_{\text{net}} = W_{12} + W_{23} + W_{31}$$

$$= 194.4 + 4486 - 554.517$$

$$[W_{\text{net}} = 125.88 \text{ KJ}] \quad \underline{\text{Ans}}$$

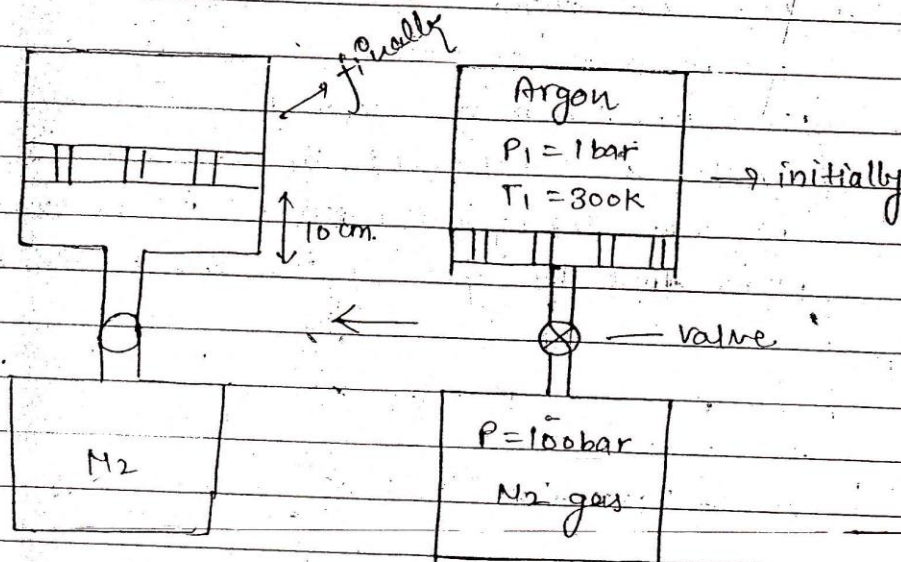
Remark ① Net work in a cycle is equal to area of closed region.

② All clockwise cycles on P-V diagram are power producing or work developing cycles. All anticlockwise cycles are power absorbing or work absorbing cycles.

Question (2) → An insulated vertical cylinder contains 0.1 kg. of Ar gas, with the help of a frictionless non-conducting piston as shown in fig. The mass of the piston is 5 kg and it initially rests on the bottom of the cylinder. Cylinder is connected to a N_2 tank at 100 bar ^{through} a pipe line fitted in ~~valve~~ The ~~see~~ valve is opened and N_2 enters the cylinder. During this process the

piston is lifted to a height of 10cm. by N_2 gas. ²⁸ (19)
The initial temp. and pressure of Argon gas are 300K and 1 bar. The final temp. of Argon is 320 K. For Argon gas take $R = 0.208 \text{ kJ/kg-K}$ and $\gamma = 1.67$.
Find (i) Work done by Argon gas
(ii) Work done by N_2 gas.

Solution $\rightarrow$



As the cylinder is insulated and piston is non-conducting and frictionless, argon gas will undergo reversible adiabatic process. Therefore

$$W = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1} = \frac{mR(T_1 - T_2)}{\gamma - 1}$$

$$W_{\text{argon}} = \frac{0.1 \times 0.208 (300 - 320)}{1.67 - 1}$$

$$W_{\text{argon}} = -0.6208 \text{ kJ}$$

(ii)

Work done by N_2 = Workdone in compressing the argon + work done in raising the piston

$$W_{N_2} = 0.6208 \text{ kJ} + \left(\frac{mgh}{1000} \right) \text{ kJ}$$

$$= 0.6208 \text{ kJ} + \left(\frac{5 \times 9.81 \times 0.1}{1000} \right) \text{ kJ}$$

$$\{W_{N_2} = 0.6257 \text{ kJ}\} \text{ Ans}$$

Question (B) → An engine cylinder has a piston of area 0.12 m^2 and contains a gas at a pressure of 1500 kPa . The gas expands according to a process which is represented by a state line on $P-V$ diagram. The final pressure is 150 kPa . Calculate work done by gas if the stroke length is 0.3 m .

Solution

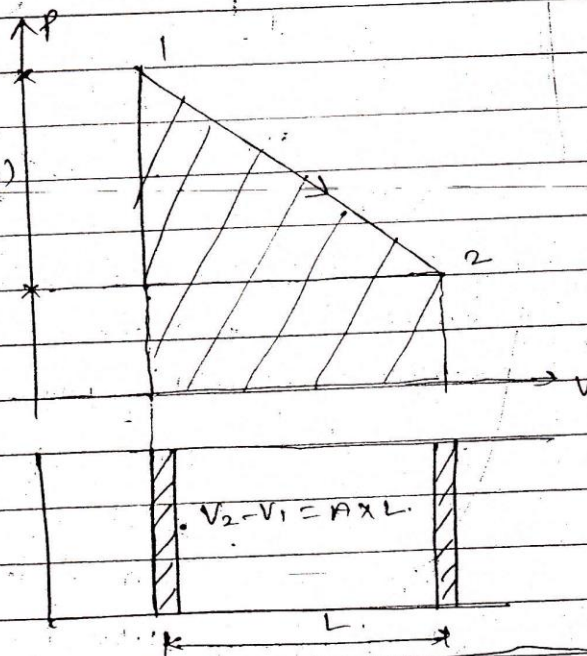
$W = \text{Area}$

$$W = \frac{1}{2} [1500 + 150] (V_2 - V_1)$$

$$W = \frac{1}{2} \times 1650 \times (0.12 \times 0.3) \quad P_1 = 1500$$

$$W = 29.7 \text{ kJ}$$

Ans



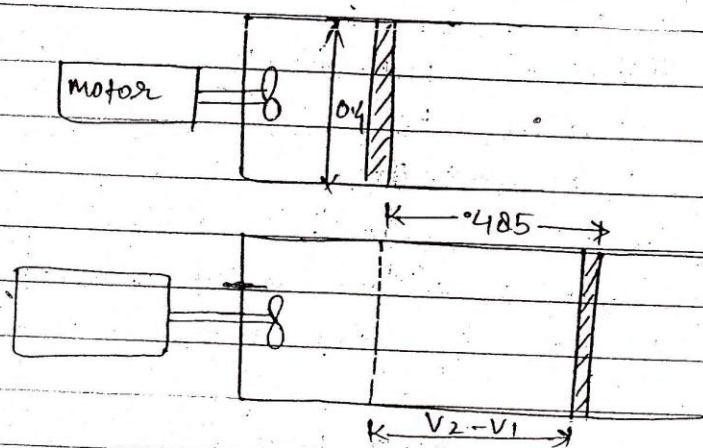
M.M.IMP

[IES-2012] A fluid contained in a horizontal cylinder fitted with a frictionless piston is continuously mixed by means of stirrer. The cylinder diameter is 0.4 m . During stirring process lasting for 10 minutes the piston slowly moves out a distance

of 0.485 m^3 at a constant atm. pressure of 101.325 kPa . The net work done by the fluid is 2 kJ . The speed of the motor driving the stirrer is 840 rpm . Determine —

- (i) Power of the motor
- (ii) Torque in the shaft.

Solution $\rightarrow$:-



$$V_2 - V_1 = \frac{\pi}{4} (0.4)^2 \times 0.485$$

$$V_2 - V_1 = 0.0609 \text{ m}^3$$

$$W_{\text{net}} = W_{\text{stirring}} + W_{\text{piston}}$$

$$2 \text{ kJ} = W_{\text{stirr}} + P_{\text{atm}} (V_2 - V_1)$$

$$2 \text{ kJ} = W_{\text{stirr}} + 101.325 (0.0609)$$

$$W_{\text{stirring}} = -4.17 \text{ kJ}$$

$$\Rightarrow W_{\text{motor}} = -W_{\text{stirring}}$$

$$W_{\text{motor}} = 4.175 \text{ kJ} = 4175 \text{ J}$$

$$[\text{Power}]_{\text{motor}} = \frac{W_{\text{motor}}}{\text{time}} = \frac{4175}{10 \times 60 \text{ sec}}$$

Eduwrite

$$P_{\text{motor}} = 6.95 \text{ watt}$$

Ans

(ii) we know $P = \frac{2\pi NT}{60}$

$$T = 0.0791 \text{ Nm}$$

* [HEAT] *

The energy transfer by virtue of temperature difference is known as heat transfer.
we know that

$$Q \propto m \quad \text{and} \quad Q \propto \Delta t$$

$$\therefore Q \propto m \cdot \Delta t$$

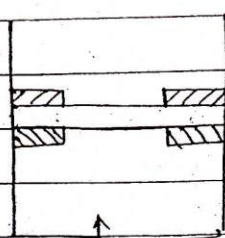
$$\text{OR} \quad Q = m \cdot c \cdot \Delta t$$

c : constant $\Rightarrow$ specific heat of substance

* Specific heat is defined as the amount of heat required to raise the temperature of unit mass of substance through unit degree temp difference.

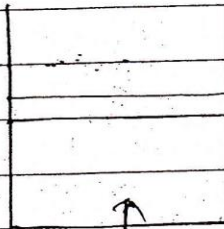
* The unit of specific heat is KJ/kg-K .

[NOTE] Solids and liquids have one specific heat whereas gases have two specific heats (c_p & c_v).



t_i
 t_f
 $w=0$
 Q_v

①



t_i
 t_f
 $w \neq 0$
 Q_p
 $w = P(V_2 - V_1)$

②

Case ① $\rightarrow$ Vol. is constant
 $w=0$

for 1a2 & 2c1.

$$\oint Q = \oint W$$

$$(\oint Q)_{1a2} + (\oint Q)_{2c1} = (\oint W)_{1a2} + (\oint W)_{2c1}$$

$$\text{Eqn ①} - \text{Eqn ②}$$

②

$$(\oint Q)_{2b1} - (\oint Q)_{2c1} = (\oint W)_{2b1} - (\oint W)_{2c1}$$

because work transfer is a path function.

$$\Rightarrow (\oint W)_{2b1} \neq (\oint W)_{2c1}$$

$$\Rightarrow (\oint W)_{2b1} - (\oint W)_{2c1} \neq 0$$

$$\Rightarrow (\oint Q)_{2b1} - (\oint Q)_{2c1} \neq 0$$

$$\Rightarrow (\oint Q)_{2b1} \neq (\oint Q)_{2c1}$$

$\Rightarrow$ Heat transfer is also a path function.

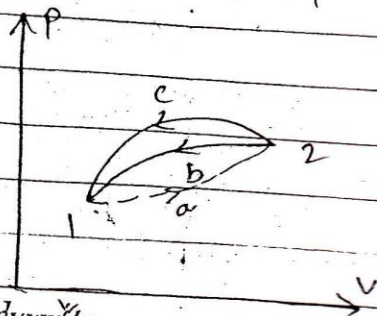
$$\text{So } dQ \Rightarrow dQ$$

$\Downarrow$

* Important points with respect to heat transfer and work transfer :-

- Both are path functions
- Both are not properties of system
- Both are inexact differentials
- Both are boundary phenomenon

(2) [Second consequence] $\rightarrow$ Energy is a property



from the first
con.
we have

$$(dQ)_{2b1} - (dQ)_{2c1} = (dw)_{2b1} - (dw)_{2c1} \quad (17)$$

$$(dQ)_{2b1} - (dw)_{2b1} = (dQ)_{2c1} - (dw)_{2c1}$$

$$(dQ - dw)_{2b1} = (dQ - dw)_{2c1}$$

The quantity $(dQ - dw)$ is same for paths b and c, i.e. this quantity is independent of path and it depends only on end points. Therefore $(dQ - dw)$ must be a property.

$$\text{Let's say } (dQ - dw)_{2b1} = dE$$

$$\text{OR } [dQ = dE + dw]_{2b1}$$

2b1 is a process.

Hence

$$dQ = dE + dw$$

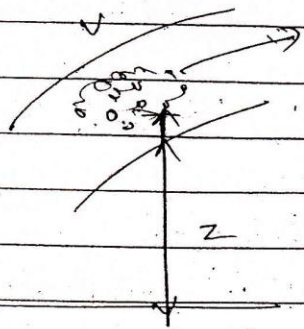
→ First law of T.D. for a closed system under going any (Rev & Irrev) process.

NOTE → The equation $dQ = dE + PdV$ is the first law of thermodynamics for a closed system undergoing reversible process only. Because

$dw = PdV$ is valid only for a reversible process.

$$E = \underbrace{K.E + P.E}_{\text{macroscopic energies}} + \underbrace{U}_{\text{microscopic energy}}$$

macroscopic energies microscopic energy



$$E = KE + PE + U$$

$$dE = d(KE) + d(PE) + dU$$

Generally K.E change & P.E change are very small.

$$dE \approx dU$$

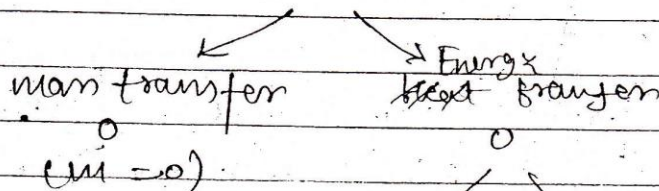
$$\delta Q = \delta W + dE$$

$$\boxed{\delta Q = dU + \delta W}$$

* This is the 1st law of ~~thermodynamics~~ T.D. equation for a closed system undergoing any process when kinetic & potential energy changes are neglected.

[3] THIRD CONS. → Energy of an isolated system is always constant

* we know for an isolated system



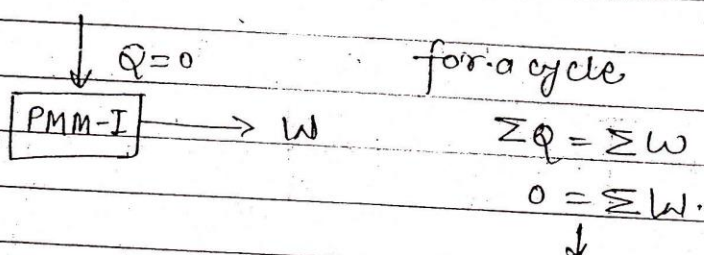
$$\text{So } \delta Q = \delta W + dE$$

$$0 = 0 + dE$$

$$dE = 0$$

$$\boxed{E = \text{constant}}$$

- 10) [4th Consequence] $\rightarrow$:- Perpetual motion machine (10)
 of first kind & i.e. (PMM-I) is impossible. 24
 * There can be no machine which develops work continuously without absorbing some other form of energy. If PMM-I is developed then it would violate first law of thermodynamics.



Violates I law of T.D.

* INTERNAL ENERGY (U): The energy associated with molecules is known as internal energy. Internal energy is an extensive property. It is generally expressed in KJ. Specific internal energy

$$u = \frac{U}{m} = \frac{\text{Total I.E.}}{\text{mass}}$$

$$u = \frac{U}{m} \rightarrow \text{ext} \rightarrow \text{ext}$$

$$\left\{ u = \frac{\text{extensive}}{m} = \text{intensive} \right\}$$

u is expressed in KJ/kg.

* Enthalpy (H) $\rightarrow$:- In thermodynamics the term "(U+PV)" appears frequently and for convenience this term is taken as enthalpy. Enthalpy is an extensive property.

It is expressed in KJ.

specific enthalpy: $h = \frac{H}{m}$ is an intensive property

h is generally expressed in KJ/kg

As $h = H/m$

$$h = \frac{U + PV}{m}$$

$$h = \left(\frac{U}{m}\right) + P\left(\frac{V}{m}\right)$$

$$h = u + Pv$$

* Heat transfer in various non-flow/closed system processes $\rightarrow$:

① [Constant volume process]

for a process

$$dQ = dU + dW$$

at constant volume $dW = 0$

$$\checkmark \left[dQ \right]_v = dU \rightarrow \text{valid for any substance}$$

$\hookrightarrow$ Heat transfer in a constant volume closed system process is equal to change in internal energy.

② [Constant pressure process]

$$dQ = dU + dW$$

$$dQ = dU + PdV$$

as pressure is constant

$$PdV \rightarrow d(PV)$$

$$(dQ)_p = dU + d(PV)$$

$$(dQ)_p = d[U + PV]$$

$$[dQ]_p = dH \rightarrow \text{Valid for closed systems reversible process for any substance}$$

* * Heat transfer in constant pressure closed system process is equal to change in enthalpy

* for an ideal gas $\Rightarrow dU = mC_v dt$ $\Rightarrow dH = mC_p dt$ $U = f(T)$	valid
--	-------

* To show that $C_p - C_v = R$ for an ideal gas $\rightarrow$

we know $H = U + PV$

or $dH = dU + d(PV) \rightarrow \text{①}$

we know, for an ideal gas

$$dH = mC_p dt, \quad dU = mC_v dt$$

$$\text{and } P = mRT$$

$\Rightarrow$ from ①

$$mC_p dT = mC_v dT + d(mRT)$$

$$mC_p dT = mC_v dT + mRdT$$

$$C_p = C_v + R$$

$$C_p - C_v = R$$

proved

$$* \quad C_p - C_v = R \quad (\text{Proved earlier})$$

$$C_p / C_v = \gamma \quad (\text{we know})$$

$$C_p = \gamma C_v$$

$$\Rightarrow C_p - C_v = R \Rightarrow \gamma C_v - C_v = R$$

$$\boxed{C_v = \frac{R}{\gamma - 1}} \rightarrow \boxed{C_p = \gamma C_v = \frac{\gamma R}{\gamma - 1}}$$

* For air

$$C_p = 1.005 \text{ kJ/kg}\cdot\text{K}$$

$$C_v = 0.718 \text{ kJ/kg}\cdot\text{K}$$

$$R = 0.287 \text{ kJ/kg}\cdot\text{K}$$

$$\gamma = 1.4$$

(3) [Heat transfer in isothermal process]

$$dQ = du + dw$$

if T is constant -

for an ideal gas $u = f(T)$

$$u = \text{constant} \quad [\text{as } T = C]$$

$$du = 0 \rightarrow \text{for ideal gas}$$

$$dQ = 0 + dw$$

$$\boxed{dQ = dw} \rightarrow \text{for ideal gas only, closed system}$$

for an ideal gas undergoing isothermal process heat transfer is equal to work transfer

(4) [Heat transfer in adiabatic process] 223 (20)

$dQ = 0$ [Because there is no heat transfer in adiabatic process]

(5) [Heat transfer in polytropic process]

$$dQ = dU + dW \quad \text{--- (1)}$$

for ideal gas $dU = m C_v dT$

$$dU = m C_v (T_2 - T_1)$$

from (1) and $dW = \frac{1}{n-1} (P_1 V_1 - P_2 V_2)$

from (1)

$$dQ = \frac{1}{(n-1)} (P_1 V_1 - P_2 V_2) + m C_v (T_2 - T_1)$$

$$dQ = \frac{1}{(n-1)} (P_1 V_1 - P_2 V_2) + m \left(\frac{R}{\gamma-1} \right) (T_2 - T_1)$$

$$dQ = \frac{1}{(\gamma-1)} m R (T_2 - T_1) + \frac{1}{(n-1)} (P_1 V_1 - P_2 V_2)$$

$$dQ = \frac{1}{\gamma-1} (m R T_2 - m R T_1) + \frac{P_1 V_1 - P_2 V_2}{(n-1)}$$

$$= \frac{P_2 V_2 - P_1 V_1}{\gamma-1} + \frac{P_1 V_1 - P_2 V_2}{(n-1)}$$

$$dQ = (P_1 V_1 - P_2 V_2) \left[\frac{1}{n-1} - \frac{1}{\gamma-1} \right]$$

$$dQ = (P_1 V_1 - P_2 V_2) \cdot \frac{(\gamma-1)(n-1)}{(\gamma-1)(n-1)}$$

$$dQ = \frac{(P_1 V_1 - P_2 V_2)}{(n-1)} \left[\frac{\gamma-n}{\gamma-1} \right]$$

or $dQ = \frac{\gamma-n}{\gamma-1} \left[W_{\text{Polytropic}} \right]$

* [POLYTROPIC SPECIFIC HEAT] : (C_{poly})
for a polytropic process

$$dQ_{poly} = \left(\frac{\gamma - n}{\gamma - 1} \right) W_{poly}$$

$$dQ_{poly} = \left(\frac{\gamma - n}{\gamma - 1} \right) \left(\frac{P_1 V_1 - P_2 V_2}{n - 1} \right)$$

$$= \frac{\gamma - n}{\gamma - 1} \left[\frac{m R T_1 - m R T_2}{n - 1} \right]$$

$$= \frac{\gamma - n}{\gamma - 1} \frac{m R (T_1 - T_2)}{(n - 1)}$$

$$= \frac{\gamma - n}{n - 1} \cdot \left(\frac{m R}{\gamma - 1} \right) (T_1 - T_2)$$

$$= \left(\frac{\gamma - n}{n - 1} \right) \cdot m \left(\frac{R}{\gamma - 1} \right) (T_1 - T_2)$$

$$= \left(\frac{\gamma - n}{n - 1} \right) m C_v (-dT)$$

$$dQ_{poly} = \left(\frac{n - \gamma}{n - 1} \right) m C_v dT$$

$$dQ_{poly} = m \left[\frac{n - \gamma}{n - 1} C_v \right] dT$$

C_{poly}

$$C_{poly} = \left(\frac{n - \gamma}{n - 1} \right) C_v$$

we know $1 < n < \gamma$

$$\rightarrow \{ C_{poly} < 0 \}$$

$$(dQ)_{\text{poly}} = \left(\frac{\gamma - n}{\gamma - 1} \right) W_{\text{poly}}$$

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let take $\gamma = 1.4$, $n = 1.2$

$$(dQ)_{\text{poly}} = \left(\frac{1.4 - 1.2}{1.4 - 1} \right) W_{\text{poly}}$$

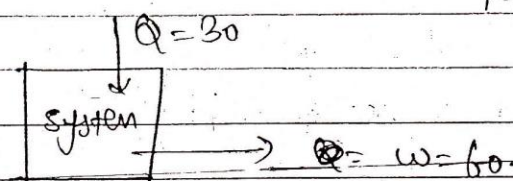
$$(dQ)_{\text{poly}} = \frac{0.2}{0.4} \times W_{\text{poly}}$$

$$(dQ)_{\text{poly}} = \frac{1}{2} W_{\text{poly}}$$

OR $W_{\text{poly}} = 2 (dQ)_{\text{poly}}$

suppose $(dQ)_{\text{poly}} = 30 \text{ kJ}$

$$W_{\text{poly}} = 60 \text{ kJ}$$



That is the reason behind -ve C_{poly} extra work is provided by

for $1 < n < \gamma$, $C_{\text{poly}} < 0$, i.e. in such a

polytropic process though heat is supplied, there is a decrease in temp. This is because work transfer is more than heat transfer. And this excess work transfer comes from the internal energy of the system.

And hence as there is a decrease in internal energy, there is a decrease in temperature.

** Show that $PV^\gamma = \text{constant}$ for an adiabatic process $\rightarrow$

We know $dQ = dU + PdV \rightarrow$ for a reversible process.

for ideal gas, undergoing adiabatic process

$$dU = mC_v dT \quad \& \quad dQ = 0$$

$$0 = mC_v dT + PdV$$

$$\text{OR } PdV = -mC_v dT \quad \text{--- (1)}$$

Now considering

$$H = U + PV$$

$$dH = dU + PdV + VdP$$

$$\text{from (1)} \quad dH = dQ + VdP$$

for ideal gas & adiabatic process

$$dH = mC_p dT$$

$$dQ = 0$$

$$mC_p dT = 0 + VdP$$

$$m VdP = mC_p dT \quad \text{--- (3)}$$

$$\frac{Eq (3)}{Eq (1)} \Rightarrow \frac{VdP}{PdV} = - \frac{mC_p dT}{mC_v dT}$$

$$\frac{V}{dV} \cdot \frac{dP}{P} = -\gamma$$

$$\frac{dP}{P} = -\gamma \frac{dV}{V}$$

$$\frac{dP}{P} + r \frac{dV}{V} = 0$$

integrating

$$\ln P + r \ln V = \ln C$$

$$\ln P + \ln (V)^r = \ln C$$

$$PV^r = C$$

valid for reversible adiabatic process.

valid for both

open closed

Imp ***

NOTE

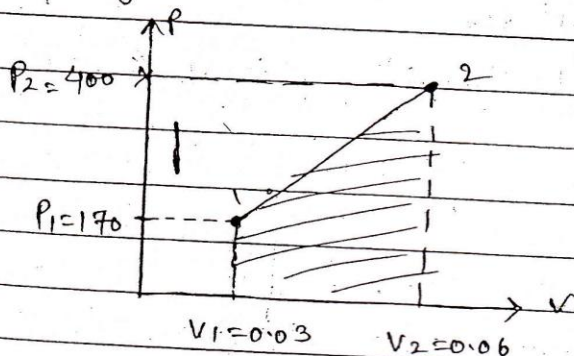
The eqn $PV^r = C$ is valid for an ideal gas undergoing reversible adiabatic process.

[CONVENTIONAL QUESTIONS]

Question 1 (IAS) → A fluid is contained in a cylinder for a spring loaded frictionless piston so that pressure in the fluid is a linear function of volume i.e. $P = a + bV$, a & b are constants.

Internal energy of the fluid is given by $U = 34 + 3.15PV$, where U is in kJ, P is kPa and V in m^3 . If the fluid changes from initial state of $P_1 = 170$ kPa, $V_1 = 0.03$ m^3 to a final state of $P_2 = 400$ kPa, $V_2 = 0.06$ m^3 , then find the magnitude and direction of heat and work transfer?

Soln



$W = \text{area}$

$$= \frac{1}{2} (170 + 400) (0.06 - 0.03)$$

$$= \frac{570}{2} \times 0.03$$

$$W = 8.55 \text{ kJ}$$

Ans

$$dQ = dU + dW$$

$$dU = U_2 - U_1$$

$$U_2 = 34 + 3.15 P_2 V_2$$

$$U_1 = 34 + 3.15 P_1 V_1$$

$$dU = U_2 - U_1 = 3.15 (P_2 V_2 - P_1 V_1)$$

$$dU = 3.15 [400 \times 0.06 - 170 \times 0.03]$$

$$dU = 59.535 \text{ kJ}$$

$$\Rightarrow dQ = 59.535 + 8.55$$

$$dQ = 68.08 \text{ kJ}$$

Both heat & work transfer are +ve
that means heat is added to system and
work is done by the system.

Imp $\Rightarrow$

An insulated rigid pressure vessel is divided into two portions by a partition. The first part of the vessel is occupied by an ideal gas at a pressure P_1 , volume V_1 and temp T_1 . The other part is occupied by same ideal gas but at a pressure P_2 , vol V_2 and temp T_2 . The partition is removed and two portions mixed with each other. Show that the final pressure P_3 and the final temp T_3 are given by

$$P_3 = \frac{P_1 V_1 + P_2 V_2}{V_1 + V_2}$$

$$T_3 = \frac{\frac{P_1 V_1}{T_1} + \frac{P_2 V_2}{T_2}}{\frac{P_1 V_1}{T_1} + \frac{P_2 V_2}{T_2}}$$

solution >

P_1	P_2
V_1, m_1	V_2, m_2
T_1	T_2

initial condition

P_3, V_3, T_3
$m_3 = m_1 + m_2$
$V_3 = V_1 + V_2$

final condition

as $m_1 + m_2 = m_3$ (mass conservation)

$$\frac{P_1 V_1}{RT_1} + \frac{P_2 V_2}{RT_2} = \frac{P_3 V_3}{RT_3}$$

$$\frac{P_3 V_3}{T_3} = \frac{P_1 V_1}{T_1} + \frac{P_2 V_2}{T_2}$$

$$T_3 = \frac{P_3 V_3}{\frac{P_1 V_1}{T_1} + \frac{P_2 V_2}{T_2}} \quad \text{--- (1)}$$

I Law of T.D

$$dQ = dU + dW$$

as it is adiabatic & constant vol process

$$dQ = 0, dW = 0$$

$$dU = 0$$

$$\text{i.e. } U_f = U_i$$

$$\text{but } U_f = U_3 \text{ \& } U_i = U_1 + U_2$$

$$\Rightarrow U_3 = U_1 + U_2$$

$$m_3 C_v T_3 = m_1 C_v T_1 + m_2 C_v T_2$$

$$m_3 T_3 = m_1 T_1 + m_2 T_2$$

$$m_3 R T_3 = m_1 R T_1 + m_2 R T_2$$

$$\{ P_3 V_3 = P_1 V_1 + P_2 V_2 \} \quad \text{--- (2)}$$

$$P_3 = \frac{P_1 V_1 + P_2 V_2}{V_3}$$

$$P_3 = \frac{P_1 V_1 + P_2 V_2}{V_1 + V_2}$$

from ① and ②

$$T_3 = \frac{P_1 V_1 + P_2 V_2}{\frac{P_1 V_1}{T_1} + \frac{P_2 V_2}{T_2}}$$

** Question) A fluid system containing a piston-cylinder machine, passes through a complete cycle of 4 processes. The summation of all heat transfers is -340 kJ/cycle . The system completes 200 cycles per minutes. Complete the following table and also calculate network in kW.

Process	$Q \text{ (kJ/min)}$	$W \text{ (kJ/min)}$	$U \text{ (kJ/min)}$
1-2	0	4340	-4340
2-3	42000	0	42000
3-4	-4200	69000	-73200
4-1	-105800	-141340	85540

Given $\Sigma Q = -340 \frac{\text{kJ}}{\text{min} \cdot \text{cycle}}$

$$= -340 \frac{\text{kJ}}{\frac{1}{200} \text{ min}}$$

$$= -340 \times 200 \text{ kJ/min}$$

$$\Sigma Q = -68000 \text{ kJ/min.}$$

22/20

$$\Sigma Q = Q_{12} + Q_{23} + Q_{34} + Q_{41}$$

$$Q_{41} = -68000 + 4200 - 42000 - 0$$

$$Q_{41} = -105800 \text{ kJ/min.} \quad \text{A}$$

for process 1-2

$$dQ_{1-2} = dU_{1-2} + dW.$$

$$0 = dU_{1-2} + 4340.$$

$$\{dU_{1-2} = -4340 \text{ kJ/min}\} \quad \text{A}$$

for process 2-3

$$dQ_{2-3} = dU_{2-3} + dW_{2-3}$$

$$dU_{2-3} = 42000 - 0$$

$$\{dU_{2-3} = 42000 \text{ kJ/min.}\} \quad \text{A}$$

for process 3-4

$$dW_{3-4} = dQ_{3-4} - dU_{3-4}$$

$$= -4200 - (-73200)$$

$$= -4200 + 73200$$

$$\{dW_{3-4} = 69000 \text{ kJ/min}\} \quad \text{A}$$

for a cycle

we know

$$\delta Q = \delta W$$

$$Q_{1-2} + Q_{2-3} + Q_{3-4} + Q_{4-1}$$

$$= W_{1-2} + W_{2-3} + W_{3-4} + W_{4-1}$$

$$\Rightarrow \{W_{4-1} = -141340 \text{ kJ/min}\} \quad \text{A}$$

for process 4-1

$$dQ_{4-1} = dU_{4-1} + dW_{4-1}$$

$$-105800 = dU_{4-1} + (-141340)$$

$$\{dU_{4-1} = 35540 \text{ kJ/min}\} \quad \text{A}$$

Eduwritte

$$w_{\text{net}} = \sum w$$

$$\text{but for a cycle } \sum w = \sum Q = -68000 \frac{\text{kJ}}{\text{min}}$$

$$\Rightarrow w_{\text{net}} = -68000 \frac{\text{kJ}}{\text{min}}$$

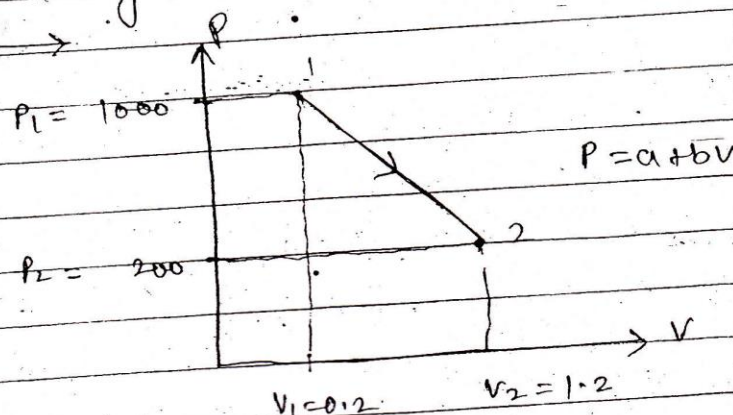
$$= -68000 \frac{\text{kJ}}{60 \text{ sec.}}$$

$$w_{\text{net}} = -1133.33 \text{ kW}$$

Ans

Question 4 (IES-2012) $\rightarrow$ A gas of mass 1.5 kg undergoes a reversible expansion process, which follows the relationship $P = a + bV$, a & b are constants. The initial and final pressures are 1000 kPa and 200 kPa respectively and corresponding volumes $V_1 = 0.2 \text{ m}^3$ and $V_2 = 1.2 \text{ m}^3$. The specific internal energy is given by $u = (1.5Pv - 85) \text{ kJ/kg}$, where P is in kPa and v in m^3/kg . Calculate the heat transfer and maximum internal energy of the gas.

<Solution>



12:00

PAGE:

DATE: / /

$w = \text{area under } P-V \text{ diagram}$

$$w = \frac{1}{2} (1000 + 200) (1.2 - 1.2)$$

$$w = 600 \text{ KJ}$$

Given

$$u = (1.5 P v - 0.5) \text{ KJ/kg.}$$

$$u = \frac{U}{m} \Rightarrow U = u \times m$$

$$U = m [1.5 P v - 0.5]$$

$$U = [1.5 P (m v) - 0.5 m] \quad [m v = V]$$

$$U = [1.5 P V - 0.5 m]$$

$$dU = U_2 - U_1$$

$$= [1.5 P_2 V_2 - 0.5 m - 1.5 P_1 V_1 + 0.5 m]$$

$$dU = 1.5 (P_2 V_2 - P_1 V_1)$$

$$dU = 1.5 [200 \times 1.2 - 1000 \times 0.2]$$

$$dU = 60 \text{ KJ}$$

$$dQ = dU + dw$$

$$dQ = 60 + 600$$

$$dQ = 660 \text{ KJ} \text{ Ans.}$$

for max internal energy $\rightarrow$

$$\text{we have } U = 1.5 P v - 0.5 m$$

$$U = 1.5 P v - 127.5$$

$$\text{as } P = a + b v$$

$$U = 1.5 (a + b v) v - 127.5$$

$$U = 1.5 (a v + b v^2) - 127.5$$

$$\text{for max } U \rightarrow \frac{dU}{dV} = 0$$

$$\frac{dU}{dV} = 1.5 [a(1) + b(2V)] - 0 = 0$$

$$a + 2bV = 0$$

$$\left\{ V = -a/2b \right\}$$

$$\text{as } P = a + bV$$

$$P_1 = a + bV_1$$

$$P_2 = a + bV_2$$

$$1000 = a + b(1.2)$$

$$200 = a + b(1.2)$$

$$800 = b(-1)$$

$$b = -800 \Rightarrow a = 1160$$

$$\text{so hence } V = -\frac{a}{2b} = -\frac{1160}{2 \times -800}$$

$$[V = 0.725]$$

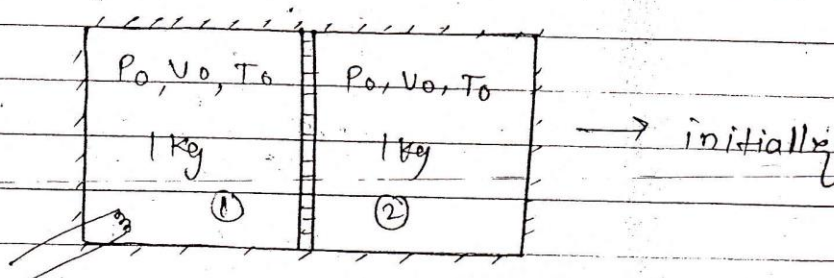
$$U_{\max} = [1.5(aV + bV^2) - 127.5]_{V=0.725}$$

$$U_{\max} = 1.5(1160 \times 0.725 + 800 \times 0.725^2) - 127.5$$

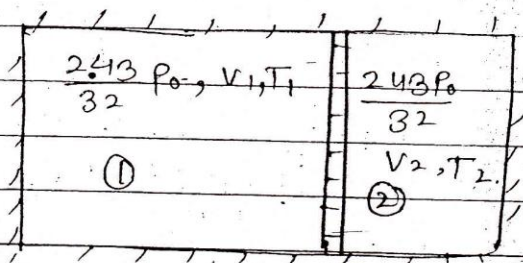
$$= 1.5(\checkmark)$$

$$[U_{\max} = 503.25 \text{ kJ}] \quad \underline{\underline{Ans}}$$

Question A rectangular box as shown in figure 229 (26) has a partition which slides without friction along the length of box. Initially the two chambers of box has 1 kg. of each of an ideal gas ($\gamma = 5/3$) at a pressure P_0 , vol V_0 & temp T_0 . The chamber on left is heated with the help of electric heater. The walls of the box and partition are thermally insulated. The gas in the left chamber expands pushing the partition until the final pressure in both chamber is $\frac{243}{32} P_0$. Determine the final temp of gas in each chamber in terms of T_0 and also calculate workdone by the gas in right chamber.



Soln $\rightarrow$ finally



initial volume = final vol.

$$V_0 + V_0 = V_1 + V_2$$

$$V_1 + V_2 = 2V_0 \quad \text{--- (1)}$$

from mass conservation -

$$(m) \quad P$$

$$PV = nRT$$

$$n = \frac{PV}{RT}$$

for left chamber.

$$\frac{P_0 V_0}{RT_0} = \frac{243}{32} \frac{P_0 V_1}{RT_1}$$

$$V_1 = 32 V_0 T_1$$

$$243 T_0$$

$$V_2 = \frac{32 V_0 T_2}{243 T_0}$$

$$\text{as } V_1 + V_2 = 2V_0$$

$$\frac{32 V_0}{243 T_0} (T_1 + T_2) = 2V_0$$

$$\left\{ T_1 + T_2 = \frac{243 T_0}{16} \right\} \quad \text{--- (1)}$$

* As the walls and partitions are thermally insulated the gas in the right chamber undergoes a reversible adiabatic process (partition is frictionless).

Hence applying reversible adiabatic conditions for the gas in right chamber.

$$\frac{T_f}{T_i} = \left(\frac{P_f}{P_i} \right)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{T_2}{T_0} = \left(\frac{243 P_0}{32 P_0} \right)^{\frac{5-1}{5}}$$

$$\frac{T_2}{T_0} = \left(\frac{3^5}{2^5} \right)^{2/5}$$

$$\frac{T_2}{T_0} = \frac{9}{4}$$

23

(23)

$$T_2 = 2.25 T_0 \quad \text{Ans.}$$

from (1)

$$T_1 = \frac{243 T_0}{16} - T_2$$

$$T_1 = \frac{243}{16} T_0 - \frac{9}{4} T_0$$

$$= \frac{243}{16} T_0 - \frac{36}{16} T_0$$

$$= \frac{207}{16} T_0$$

$$T_1 = \frac{207}{16} T_0 \quad \text{Ans.}$$

work done by gas in right chamber $\rightarrow$ The system is a closed system and the right side gas undergoes reversible adiabatic process.

$$\therefore W = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1} = \frac{m R T_1 - m R T_2}{\gamma - 1}$$

$$= \frac{m R}{\gamma - 1} [T_1 - T_2]$$

$$W = \frac{1 \times R [T_0 - 2.25 T_0]}{\frac{5}{3} - 1}$$

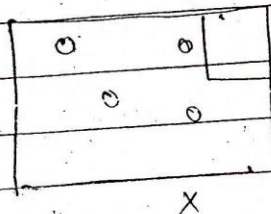
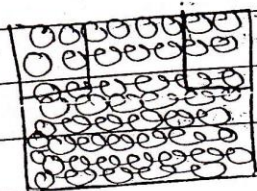
$$W = \frac{3 R}{2} [-1.25 T_0]$$

$$W = (1.5) (-1.25) R T_0$$

$$W = -1.875 R T_0 \quad \text{Ans.}$$

[Principle of continuum]

- (1) Large no. of molecules
- (2) Individual behaviour of molecules is not taken into consideration
- (3) Mean free path is lower



Q.13 (workbook)

Let's check for option (d)

→ a property is an exact differential

$$\text{let } dz = M dx + N dy$$

exact diff. if $\left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y$

$$\text{we have } \frac{dT}{T} - \frac{V}{T} dP$$

$$\Rightarrow \left(\frac{1}{T}\right) dT - \left(\frac{V}{T}\right) dP$$

$$M = \frac{1}{T}, x = T, N = \left(-\frac{V}{T}\right), y = P$$

$$\left(\frac{\partial M}{\partial y}\right)_x = \left[\frac{\partial}{\partial P} \left(\frac{1}{T}\right)\right]_T = 0 \Rightarrow (T = \text{constant})$$

$$\left(\frac{\partial N}{\partial x}\right)_y = \left[\frac{\partial}{\partial T} \left(-\frac{V}{T}\right)\right]_P$$

for ideal gas $PV = mRT$

$$\frac{V}{T} = \frac{mR}{P}$$

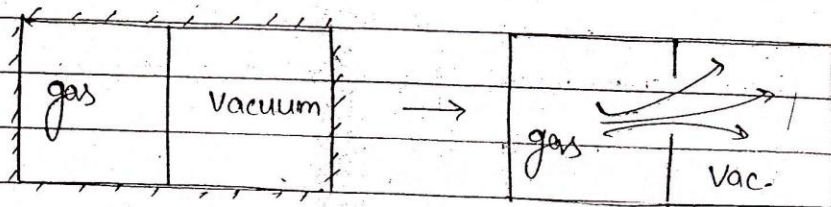
$$\left(\frac{\partial N}{\partial x}\right)_y = \left[\frac{\partial}{\partial T} \left(-\frac{mR}{P}\right)\right]_P = 0$$

it is a property

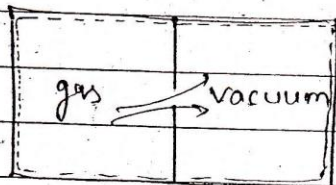
[FREE EXPANSION]

(23) (20)

The expansion of a gas against vacuum is known as free expansion or unrestrained expansion.



CASE I if gas & vacuum are taken as system



free expansion work = 0
as no effect on surrounding

$$dq = dU + dw$$

$$\downarrow \quad \quad \downarrow$$

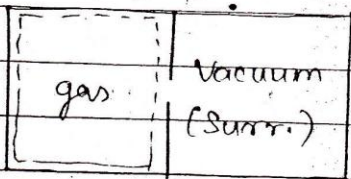
$$0 \quad \quad 0$$

$$\Rightarrow dU = 0$$

$U = \text{constant} \Rightarrow$ free expansion.

free expansion work = 0

[CASE - II] Taking gas only as system.



$\Rightarrow$ we know

$$W_{\text{sys}} + W_{\text{surr}} = 0$$

as $W_{\text{surr}} = 0$

$$W_{\text{sys}} = 0$$

Hence

free expansion work = 0 always

* [FREE EXPANSION OF AN IDEAL GAS]

* we know for any gas undergoing free expansion

* $[U_i = U_f] \rightarrow$ any gas

* for an ideal gas (undergoing)

$U = f(T)$ only.

Hence if an ideal gas is undergoing free expansion

* $U_i = U_f \Rightarrow [T_i = T_f] \rightarrow$ ideal gas

* we know $H = U + PV$ (free expⁿ)

$H = f(T) + mRT$

i.e. $[H = \phi(T)]$

but $T_i = T_f$

* $\Rightarrow [H_i = H_f] \rightarrow$ ideal gas (free expⁿ)

Imp

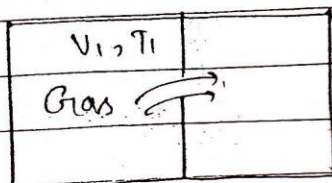
* when an ideal gas undergoes free expansion

(1) $U_i = U_f$ } for any gas

(2) $T_i = T_f$ } strictly for ideal gas

(3) $H_i = H_f$ }

*



- Before the rupture of partition, let the vol. of gas is V_1 and Temp T_1 .

after rupturing the partition, vol is V_2 and temp is same.

$\Rightarrow$ it means $dV \neq 0$.

as $V_2 \neq V_1$.

$\Rightarrow$ And gas possesses its own pressure P .

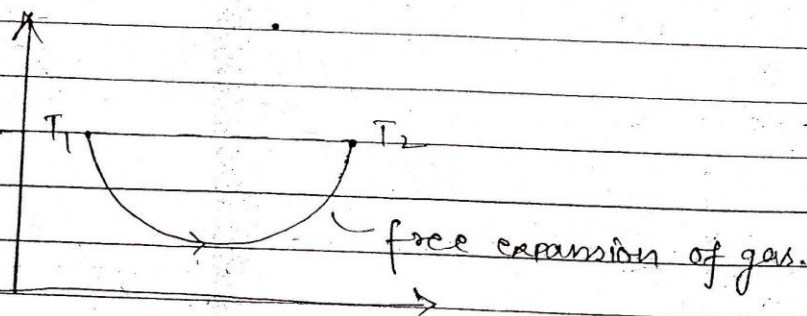
$\Rightarrow P dV \neq 0$. but work = 0.

Imp Because $P dV = \text{work}$ [only for reversible process]

* Though in free expansion of an ideal gas $T_i = T_f$, it doesn't mean that the process is isothermal, because ~~during~~ in free expansion process the temp is not constant throughout the process. Initially during expansion temp decreases and finally when molecules rub against the wall of container the temp increases.

Imp

* Though in free expⁿ $P dV$ is not equal to zero, but work transfer is equal to zero, because $P dV$ is applicable only for a reversible process, but free expⁿ is an irreversible process.



Question An ideal gas at 20 bar and 40°C is contained in a small cylinder having a volume of 15 cm^3 . This cylinder is placed inside a large container having a volume of 1500 cm^3 . The large container is perfectly insulated and evacuated. By an appropriate means the gas is allowed to discharge and fill the large container. Find the pressure after the entire assembly reaches equilibrium.

Solⁿ →

for an ideal gas

$$P_0 V = n R T$$



constant

$$P_1 V_1 = P_2 V_2$$

$$20 \times 15 = 1500 \times P_2$$

$$P_2 = 0.2 \text{ bar} \quad \text{Ans}$$

(or)

$$H_1 = H_2$$

$$U_1 + P_1 V_1 = U_2 + P_2 V_2$$

NOTE

$$P V = n R T$$



absolute pressure always

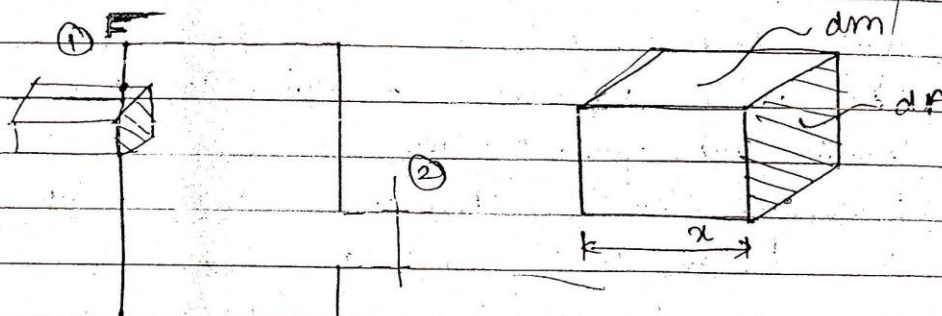
$$[P_{\text{abs}} = P_{\text{gauge}} + P_{\text{atm}}]$$

[OPEN SYSTEM]

33

30

FLOW WORK : The work transfer involved in causing the fluid element either to enter or leave the control volume is known as flow work.



$$dV = x \times dA$$

$$v = \frac{dV}{dm}$$

$$\text{flow work} = \text{force} \times \text{distance}$$

$$= P \cdot dA \times x$$

$$= P(dA \times x)$$

$$\text{flow work} = P dV$$

$$\frac{\text{flow work}}{\text{mass}} = \frac{P dV}{dm} = Pv$$

if mass 'm' enters the control volume, then

$$\text{flow work} = m(Pv)$$

$$= Pv$$

$$\text{Hence } \{ \text{flow work} = Pv \}$$

* ~~if entry~~

* If entry occurs at (1) then entry flow work is $-P_1 V_1$ & if the exit occurs at (2)

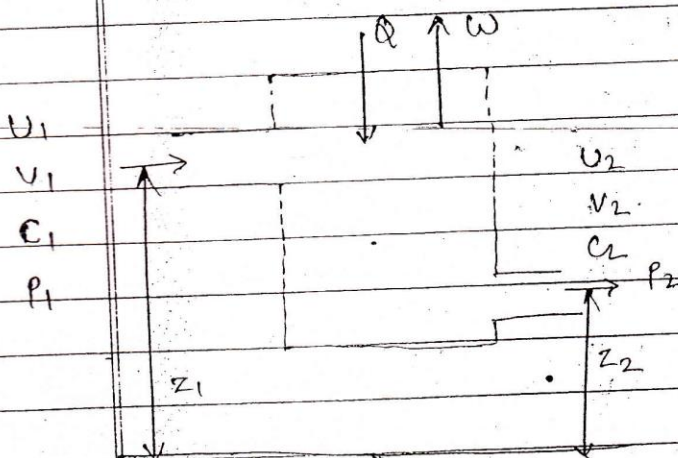
then exit flow work is $+P_2 V_2$.

M.IMP * STEADY FLOW ENERGY EQUATION * [SFEE]

* A flow is said to be steady flow if properties do not vary with respect to time at any given section. * For steady flow mass entering the control volume is equal to the mass leaving the control volume.

* Energy entering the control volume is equal to energy leaving the control volume.

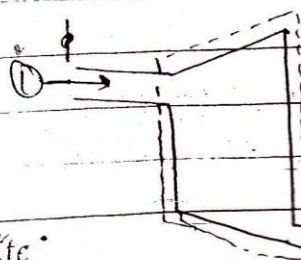
Thus for steady flow there is no accumulation of mass & energy in control volume.



for steady flow

$$E_1 = E_2$$

$$\frac{1}{2} m C^2 + m g z_1 + U_1 + Q = \frac{1}{2} m C^2 + m g z_2 + U_2 + W$$



$$W = W_{\text{entry flow}} + W_{\text{C.V.}} + W_{\text{exit flow}}$$

$$W = W_{\text{entry flow}} + W_{\text{c.v.}} + W_{\text{exit flow}} \quad (31)$$

$$\downarrow$$

$$-P_1 V_1$$

Control volume work

$$\downarrow$$

$$P_2 V_2$$

Hence SFEE

$$\frac{1}{2} m C_1^2 + mgz_1 + U_1 + Q = \frac{1}{2} m C_2^2 + mgz_2 + U_2$$

$$= P_1 V_1 + W_{\text{c.v.}} + P_2 V_2$$

$$\frac{1}{2} m C_1^2 + mgz_1 + \underbrace{(U_1 + P_1 V_1)}_{H_1} + Q = \frac{1}{2} m C_2^2 + mgz_2 + \underbrace{(U_2 + P_2 V_2)}_{H_2} + W_{\text{c.v.}}$$

$$\text{OR } \frac{1}{2} m C_1^2 + mgz_1 + H_1 + Q = \frac{1}{2} m C_2^2 + mgz_2 + H_2 + W_{\text{c.v.}}$$

for per unit mass

$$h_1 + \frac{C_1^2}{2} + gz_1 + q = h_2 + \frac{C_2^2}{2} + gz_2 + w_{\text{c.v.}}$$

↳ S.F.E.E. based on single assumption
ie steady flow.

[APPLICATIONS OF SFEE]

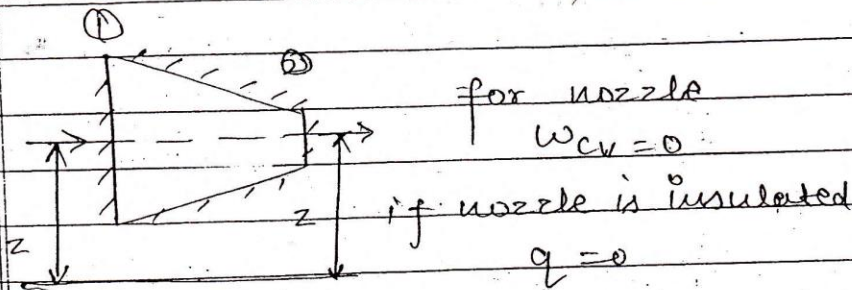
1- NOZZLE — Nozzle is a device which is used for increasing kinetic energy at the expense of pressure energy.

In 'Diffusers' K.E decreases and pressure energy increases.

* In nozzles and diffusers work transfer is equal to zero.

The fundamental SFEE is

$$h_1 + \frac{C_1^2}{2} + gz_1 + q = h_2 + \frac{C_2^2}{2} + gz_2 + w_{\text{c.v.}}$$



if $z_1 = z_2$

$$h_1 + \frac{C_1^2}{2} + z_1/g = h_2 + \frac{C_2^2}{2} + z_2/g$$

$$h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2}$$

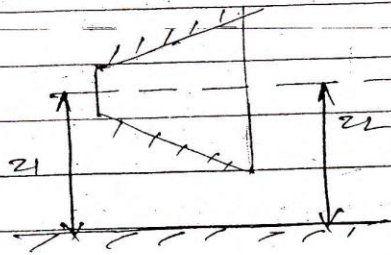
if $C_1 < C_2$

$$h_1 = h_2 + \frac{C_2^2}{2}$$

(2) TURBINE

if

- (a) steady flow
- (b) K.E changes are negligible
- (c) P.E changes are negligible
- (d) turbine is perfectly insulated then



$$h_1 + \frac{C_1^2}{2} + g z_1 + q = h_2 + \frac{C_2^2}{2} + g z_2 + w_{cv}$$

$$h_1 = h_2 + w_{cv}$$

$$w_{cv} = w_{turbine} = h_1 - h_2$$

$$\Rightarrow w_T = h_1 - h_2$$

[3] [COMPRESSOR]

236 (32)

we know

$$w_{\text{Turbine}} = h_1 - h_2 \quad \left[\begin{array}{l} \text{Turbine produces} \\ \text{work} \end{array} \right]$$

but we know, compressor absorbs work

$$\Rightarrow \boxed{w_{\text{comp.}} = h_2 - h_1}$$

This is valid under the same conditions which were in the case of turbine

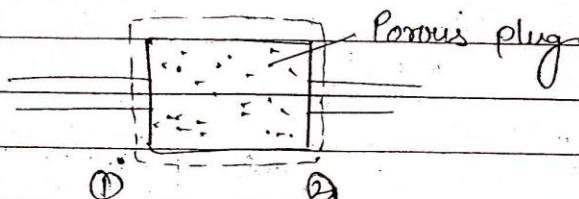
[4] [THROTTLING VALVE]

* examples of throttling -

- (a) flow through a partially opened valve
- (b) flow through a very small opening
- (c) flow through a porous plug

Characteristics of throttling -

- 1. No heat transfer
- 2. No work transfer
- 3. It is an irreversible process.
- 4. Isenthalpic process (i.e. $h_1 = h_2$)



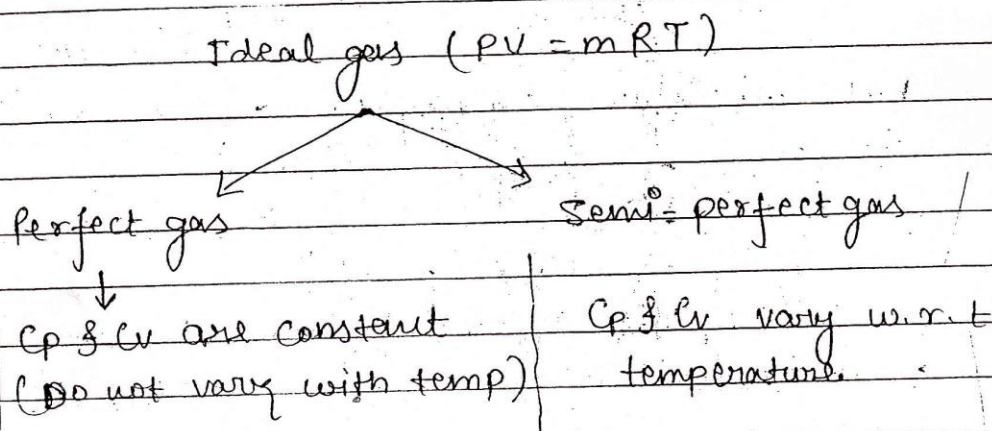
$$h_1 + \frac{c_1^2}{2} + \cancel{gz_1} + \cancel{q} = h_2 + \frac{c_2^2}{2} + \cancel{gz_2} + \cancel{w} + \cancel{q}$$

$$h_1 + \frac{c_1^2}{2} = h_2 + \frac{c_2^2}{2}$$

$$\boxed{h_1 = h_2}$$

* if K.E changes are very small

NOTE → A gas behaves as an ideal gas under low pressure & high temperature conditions.



* $PV = mRT$
 ↳ mass

we know.

$$\text{no. of moles } (n) = \frac{\text{mass}}{\text{mole weight}} = \frac{m}{M}$$

$$\Rightarrow m = nM$$

$$\Rightarrow PV = nMRT$$

$$PV = n \underbrace{(MR)} T$$

$$\boxed{PV = n \bar{R} T}$$

no. of moles.

where $\boxed{\bar{R} = MR}$

$\bar{R}$: universal gas constant $[8.314 \text{ kJ/k-mol-K}]$ Q.6

R : characteristic gas constant

Workbook

Workbook Chapter (2)

23/ (33)

Q.1 $T=C, n=70 \text{ kmol}, T=354 \text{ K}, V_2=1 \text{ m}^3$

$W_{\text{input}} = -206 \text{ kJ}$

$W_{\text{isothermal}} = P_1 V_1 \ln\left(\frac{V_2}{V_1}\right)$

$= nRT \ln\left(\frac{V_2}{V_1}\right)$

$-206 = 70 \times 8.314 \times \ln\left(\frac{V_2}{V_1}\right) \times 354$

$\ln\left(\frac{V_2}{V_1}\right) = \frac{-206}{70 \times 8.314 \times 354}$

$\frac{V_2}{V_1} = e^{(\quad)}$

$V_2 = V_1$

Q.3 →

$V=86.4 \text{ m}^3$
 $P=100 \text{ kPa}$
 $T=32^\circ \text{C}$

$dQ = dU + dW$

o (insulated)

$dW = -60 \text{ kJ/s}$ on fan is working on air (sys)

$dU = 60 \times 4 \times 3600$

$dU = 864 \text{ kJ}$

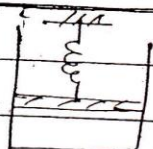
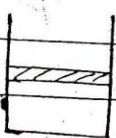
for ideal gas $dU = m c_v dT$

$m = \frac{PV}{RT} = \frac{100 \times 86.4}{0.287 \times 305} \Rightarrow m = 98.7 \text{ kg}$

here $98.7 \times 0.718 \times dT = 864 \Rightarrow dT = 12.1^\circ \text{K}$

$T_2 - T_1 = 12.1^\circ \text{C}$

Q.6 →



for work mode $= n$

$n = \text{work mode} = 1$

$n = 2$ (work mode)

Parameters req $= n+1$

$W = P(\Delta V)$

$W = P(\Delta V) + \text{spring work (Kx)}$

$\Rightarrow$

two parameter Parameter, $P, \Delta V, K$

Eduwite

$= 3$

Q.7

$Q_v = dU$ } wet know but for perfect gas $dU = m c_v dT$
 $Q_p = dH$ } $dH = m c_p dT$

PAGE: _____
 DATE: 1 / 1

Q.8

$R = mR$ $c_p = 5.19$
 $c_v = ?$

$c_p - c_v = R/m = \frac{8.314}{4}$

$c_p - 5.19 - c_v = -$

$c_v = 8.115$

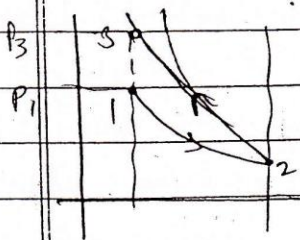
Q.14

Q is being supplied

when ice melts, volume ↓.

Piston moves down, work is done on the sys. $w < 0$

16



$w_{12} < w_{23}$

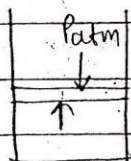
$w_{12} > 0$

$w_{23} < 0$

So $P_3 > P_1$

$w_{net} < 0$

Q.17



$P_{gas} = 300 \text{ kPa}$

$w_{total} = 300(v_2 - v_1)$

$= 300 \times 0.1$

$= 3 \text{ kJ}$

$w_{atm} = P_{atm}(\Delta v)$

$= 100 \times 0.1 = 1 \text{ kJ}$

work utilized = $3 - 1 = 2 \text{ kJ}$

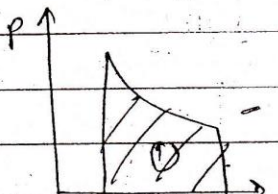
20 for isothermal

$PV = \text{const}$

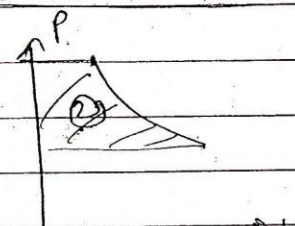
$d(PV) = 0$

$PdV + VdP = 0$

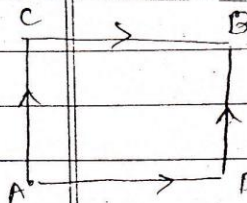
$PdV = -VdP$



$A_1 = A_2$



20



$Q_{ACB} = 180$

$w_{ACB} = 130$

$(dQ = dU + dw)_{ACB}$

$(dU)_{ACB} = 50$

$w_{ADB} = 40$

$Q_{ADB} = (dU)_{ADB} + w_{ADB}$

$(dU)_{ADB}$

$Q_{ADB} = 50 + 40 = 90$

Q.36

$T_1 = 293 \text{ K}$

$P_1 = 100$

$V_1 = 15$

$V_2 = 25$

$P_2 = 150$

$T_2 = ?$

$PV = mRT \Rightarrow m = \frac{PV}{RT}$

$m_1 = m_2$

$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$

$100 \times 15 = 150 \times 25$

$T_2 = 750$

Q.37

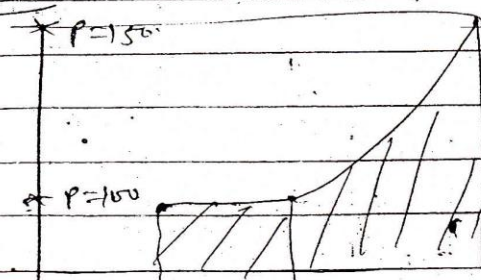
$P = 150$

$P = 100$

$V_1 = 15$

$V_2 = 20$

$V_3 = 25$



$$W_1 = (20-15) \times 100 = 500 \text{ kJ}$$

$$W_2 = \int_{20}^{25} [P_0 + 2(v-v_0)^2] v \, dv$$

$$= \int_{20}^{25} [P_0 v + 2v^2 + 2v_0^2 v - 4v^2 v_0] \, dv$$

$$= \left[P_0 v + \frac{2}{3} v^3 + \frac{2v_0^2 v}{1} - \frac{4v^2 v_0}{2} \right]_{20}^{25}$$

$$= \left[(100 \times 25) + \frac{2}{3} (25)^3 + 2(20)^2(25) - 2(25)^2(20) \right. \\ \left. - 100(20) - \frac{2}{3} (20)^3 + 2(20)^2(20) + 2(20)^2(20) \right]$$

$$= 500 + \frac{2}{3} [25^3 - 20^3] + 2 [400(5) + 2 \times 20 (20^2 - 25^2)]$$

$$= 500 + 9503.33 - 40 \times (625 - 400)$$

$$= 1083 \text{ kJ}$$

Q. 30

Remark $\rightarrow PV = mRT$ $\rightarrow P$ is always absolute pressure

initial
cond.

$$\begin{aligned} P_{\text{gauge}} &= 1 \text{ bar} \\ T &= 15^\circ\text{C} \\ &= 288 \text{ K} \\ V &= 25000 \text{ cm}^3 \end{aligned}$$

$$\begin{aligned} P_{\text{abs}} &= 1 + 1.0325 \\ &= 2.0325 \text{ bar} \end{aligned}$$

Stadium

$$t = 5^\circ\text{C}$$

$$\begin{aligned} T_2 &= 5 + 273 \\ &= 278 \text{ K} \\ t &= 15^\circ\text{C} \end{aligned}$$

for closed sys.

$$dQ = dU$$

air can be treated as ideal gas

and for ideal gas $dU = m c_v dt$

$$m = \frac{PV}{mRT} \Rightarrow \text{if } R \rightarrow \text{kJ/Kg-K then } P \text{ must be in kPa}$$

$$P = 2.0325 \text{ bar} = 2.0325 \times 100 \text{ kPa}$$

$$m = \frac{2.032 \times 100 \times 25000 \times (10^{-2})^3}{0.287 \times 288} = 6.08 \times 10^3 \text{ kg}$$

$$\text{So } dQ = dU = m c_v dt = 6.08 \times 10^3 \times 0.718 \times (278 - 288)$$

$$[dQ = -43.7 \text{ Joule}]$$

Eduwrite

(39)

$$P_1 = ?$$

$$T_1 = 200$$

$$P_2 = 1 \text{ bar (gauge)}$$

$$T_2 = 270$$

$$\frac{P_1}{P_2} = \frac{T_1}{T_2} \Rightarrow \frac{P_1}{2.013} = \frac{200}{270}$$

$$P_1 = 2.085 \text{ bar (abs)}$$

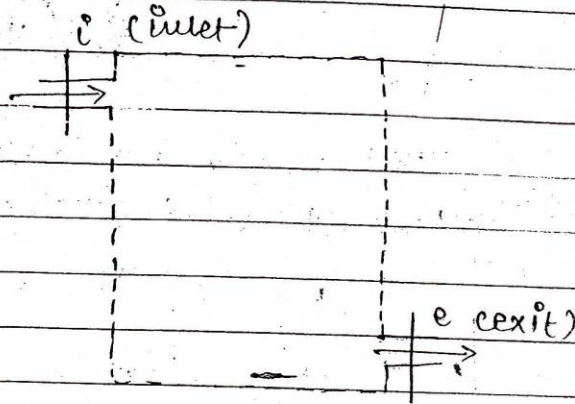
$$P_1 (\text{gauge}) = 2.085 - 1.013 = 1.07 \text{ bar.}$$

[UNSTEADY STATE FLOW]

23/35

In unsteady flow there is accumulation of mass and energy in the control volume.

Let m_i & m_e be the masses entering and leaving the control volume.



1. Conservation of mass -

$$\left(\frac{dm}{dt}\right)_{c.v.} = \frac{dm_i}{dt} - \frac{dm_e}{dt}$$

$$\boxed{\left(\frac{dm}{dt}\right)_{c.v.} = \dot{m}_i - \dot{m}_e} \quad \text{--- (1)}$$

2. Conservation of energy -

$$\left(\frac{dE}{dt}\right)_{c.v.} = \frac{dE_i}{dt} - \frac{dE_e}{dt}$$

We know S.F.E.E.

$$h_1 + \frac{C_1^2}{2} + gz_1 + q = h_2 + \frac{C_2^2}{2} + gz_2 + w$$

For unsteady flow -

$$E_i = \dot{m}_i h_i + \frac{\dot{m}_i C_i^2}{2} + \dot{m}_i g z_i + \dot{q}$$

$$E_i = \dot{m}_i h_i + \frac{\dot{m}_i C_i^2}{2} + \dot{m}_i g z_i + \dot{q}$$

$$E_e = \dot{m}_e h_e + \frac{\dot{m}_e C_e^2}{2} + \dot{m}_e g z_e + \dot{q}$$

Eduwrite

$$P_e = h_e + \frac{C_e^2}{2} + g z_e + w_{cv}$$

$$E_e = m_e h_e + \frac{m_e C_e^2}{2} + m_e g z_e + W_{cv}$$

$$\left(\frac{dE}{dt} \right)_{cv} = \frac{d}{dt} \left[m_i h_i + \frac{m_i C_i^2}{2} + m_i g z_i + \dot{Q} \right] - \frac{d}{dt} \left[m_e h_e + \frac{m_e C_e^2}{2} + m_e g z_e + W_{cv} \right]$$

Till now, no assumptions are made.
Now ~~neglecting~~ the change in K.E & P.E.

$$E = P.E + K.E + U$$

$$dE = d(P.E) + d(K.E) + dU$$

$$dE = dU$$

$$\text{so } \left[\frac{dU}{dt} \right]_{cv} = \frac{d}{dt} (m_i h_i + \dot{Q}) - \frac{d}{dt} (m_e h_e + W_{cv})$$

$$\left[\frac{dU}{dt} \right]_{cv} = \frac{d}{dt} (m_i h_i) + \dot{Q} - \frac{d}{dt} (m_e h_e) - \dot{W}_{cv}$$

Assuming the h_i & h_e are not changing with respect to time.

$$\left[\frac{dU}{dt} \right]_{cv} = h_i \frac{dm_i}{dt} + \dot{Q} - h_e \frac{dm_e}{dt} - \dot{W}_{cv}$$

$$\boxed{\left[\frac{dU}{dt} \right]_{cv} = \dot{m}_i h_i + \dot{Q} - \dot{m}_e h_e - \dot{W}_{cv}}$$

②

Assumption made in derivation -

- (1) Change in K.E & P.E is zero.
- (2) h_i & h_e are constant w.r.t. time.

Dimensions $\rightarrow$

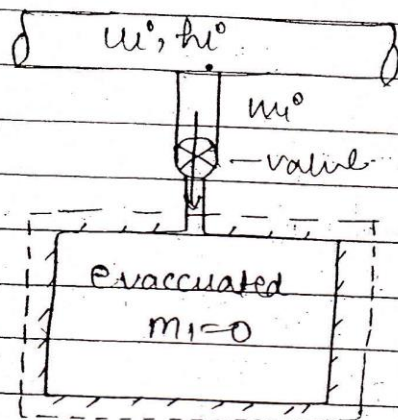
$$\dot{m}_i h_i = \frac{\text{kg}}{\text{s}} \times \frac{\text{KJ}}{\text{kg}} = \frac{\text{KJ}}{\text{sec.}}$$

So unit of $\frac{dU}{dt} \rightarrow \frac{\text{KJ}}{\text{sec.}}$

unit of U must be KJ.

Question $\rightarrow$ An insulated storage tank that initially evacuated is connected to a supply pipeline carrying a fluid at a specific internal energy u_i & sp. enthalpy h_i . The valve is opened & fluid flows into the tank from the supply pipe line & reaches the pressure same as that of the supply line. Show that the final sp. internal energy of the fluid in the tank is equal to h_i .

Soln $\rightarrow$



$$\left(\frac{dm}{dt} \right)_{\text{C.V.}} = \dot{m}_i - \dot{m}_e$$

as no mass is leaving the C.V.
 $\dot{m}_e = 0$

$$\text{So } \left(\frac{dm}{dt} \right)_{\text{C.V.}} = \dot{m}_i \quad \text{--- (1)}$$

$$\left(\frac{dU}{dt}\right)_{cv} = \dot{m}_i h_i + \dot{Q} - \dot{m}_e h_e - \dot{W}_{cv}$$

$$\dot{Q} = 0, \quad \dot{W}_{cv} = 0, \quad \dot{m}_e = 0$$

$$\left(\frac{dU}{dt}\right)_{cv} = \dot{m}_i h_i \quad \text{--- (2)}$$

Substituting (1) in (2)

$$\left(\frac{dU}{dt}\right)_{cv} = \left(\frac{dm}{dt}\right)_{cv} h_i$$

integrating

$$(dU)_{cv} = (dm)_{cv} \cdot h_i$$

$$U_2 - U_1 = (m_2 - m_1) h_i \Rightarrow m_2 U_2 - m_1 U_1 = (m_2 - m_1) h_i$$

but initially tank was evacuated

$$\text{so } m_1 = 0$$

$$m_2 U_2 = m_2 h_i$$

$$U_2 = h_i$$

mention assumption

$$(i) d(KE), d(PE) = 0$$

$$(ii) h_e, h_i \text{ are constant w.r.t time}$$

Note → if the gas is an ideal gas

$$U_2 = h_i$$

$$\text{ie } C_v T_2 = C_p T_i$$

$$T_2 = (C_p/C_v) T_i$$

$$\text{Imp } T_2 = \gamma T_i$$

<Question> [ES-2011]

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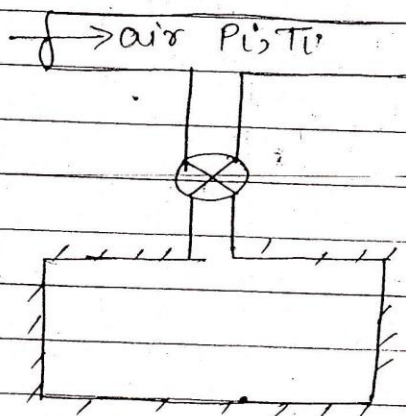
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The pressure cylinder of volume V contains air at a pressure P_1 & temp. T_1 . It is to be filled from a supply air pipe line maintained at a constant pressure P_i & temp. T_i . Show that the final temp. of air in the cylinder after it has been charged to the pressure same as the supply pressure is given by

$$T_2 = \frac{\gamma T_i}{1 + \frac{P_1}{P_i} \left(\frac{\gamma T_i}{T_1} - 1 \right)}$$

Assume the tank to be insulated.

Solution



Initially

V, T_1, P_1

$$m_1 = \frac{P_1 V}{R T_1}$$

finally

V, T_2, P_i

$$m_2 = \frac{P_i V}{R T_2}$$

from last question

$$m_2 u_2 - m_1 u_1 = (m_2 - m_1) h_u$$

$$m_2 C_v T_2 - m_1 C_v T_1 = (m_2 - m_1) C_p T_i$$

$$(m_2 T_2 - m_1 T_1) = (m_2 - m_1) \frac{C_p}{C_v} T_1$$

$$\frac{P_2 V}{R} - \frac{P_1 V}{R} = \left(\frac{P_2 V}{R T_2} - \frac{P_1 V}{R T_1} \right) \gamma T_1$$

$$(P_2 - P_1) = \left(\frac{P_2}{T_2} - \frac{P_1}{T_1} \right) \gamma T_1$$

$$P_2 - P_1 = \frac{\gamma T_1 P_2}{T_2} - \frac{\gamma T_1 P_1}{T_1}$$

$$\frac{\gamma T_1 P_2}{T_2} = P_2 - P_1 + \frac{\gamma T_1 P_1}{T_1}$$

So that

$$T_2 = \frac{\gamma T_1 P_2}{P_2 - P_1 + \frac{\gamma T_1 P_1}{T_1}}$$

$$T_2 = \frac{\gamma T_1 P_2}{P_2 \left[1 - \frac{P_1}{P_2} + \frac{\gamma T_1 P_1}{T_1 P_2} \right]}$$

$$T_2 = \frac{\gamma T_1}{1 + \frac{P_1}{P_2} \left(\frac{\gamma T_1}{T_1} - 1 \right)}$$

[SECOND LAW OF THERMODYNAMICS]

38

* DIRECTIONAL LAW *

First law of thermodynamics simply says that energy is conserved. It doesn't give any direction for a particular process. It is the second law of T.D which gives direction for a particular process through the concept of ENTROPY. And hence second law is known as directional law.

Heat is known as low grade of energy (LGE) & work is known as high grade energy (HGE). Complete conversion of LGE (heat) into HGE (work) in a cycle is impossible.

THERMAL ENERGY RESERVOIR (TER).

SOURCE → It is a reservoir which can supply thermal energy without undergoing temperature change.

SINK → It is a reservoir which absorbs thermal energy without undergoing any temp change.

[Kelvin - Plank statement]

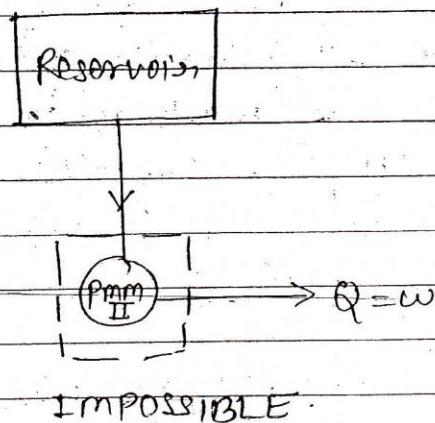
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→ STATEMENT OF 2nd LAW →

It is impossible to develop a device operating on a cycle, which produces work continuously by exchanging heat with a single reservoir.

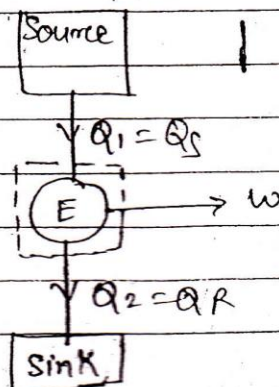
This device is known as PMM-II and its efficiency is 100%.

Therefore achieving 100% efficiency is impossible & violation of 2nd law of T.D.



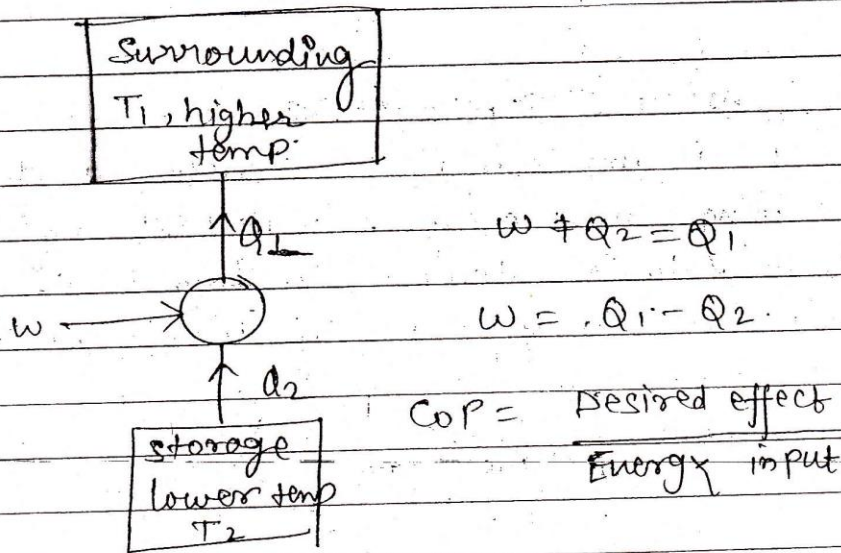
[HEAT ENGINE]

Heat engine is a device which converts part of heat into work and rejects remaining heat to sink or surrounding.



[REFRIGERATOR]

A refo is a device which maintains lower temp. compared to surroundings. As lower temp. are to be maintained continuously, therefore refrigerators operates on a cycle.



$$(COP)_R = \frac{Q_2}{W}$$

$$(COP)_R = \frac{Q_2}{Q_1 - Q_2}$$

The above equation is valid for both rev and irrev. refrigerators.

$$Q_1 = W + Q_2 \Rightarrow W = Q_1 - Q_2$$

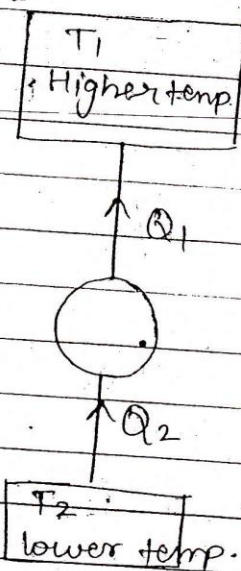
$$\eta = \text{output/input} \Rightarrow \eta = \frac{W}{Q_1}$$

$$\eta = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1} \Rightarrow \boxed{\eta = 1 - \frac{Q_R}{Q_S}}$$

This is valid for any (rev. / irrev.) heat engine. Heat engine is a logical development of Kelvin plank's statement.

CLAUSIUS STATEMENT

It is impossible to develop a device operating in a cycle, which transfer heat from lower temp. to higher temp. without any external work input.



IMPOSSIBLE

[HEAT PUMP] $\rightarrow$ Heat pump is a device which maintains higher temp. compare to surroundings.

$$Q_1 = W + Q_2$$

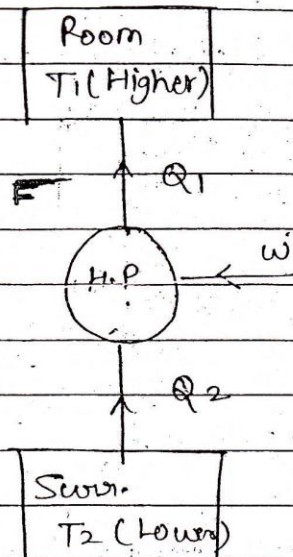
$$W = Q_1 - Q_2$$

$$COP = \frac{\text{Desired effo}}{\text{work input}}$$

$$COP = \frac{Q_1}{W}$$

$$[COP]_{H.P.} = \frac{Q_1}{Q_1 - Q_2}$$

Valid for any cycle
(eg. Rev. or Irrev.)



* Relationship b/w $(COP)_{H.P.}$ & $(COP)_{R.H.P.}$ operating b/w same temperature limits:-

$$\text{we know } (COP)_{H.P.} = \frac{Q_1}{Q_1 - Q_2}$$

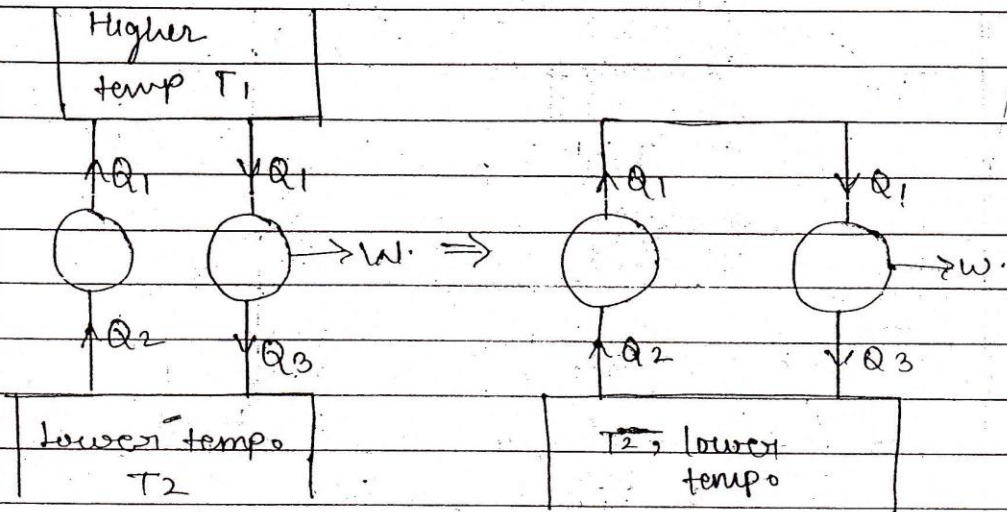
$$(COP)_{R.H.P.} = \frac{Q_2}{Q_1 - Q_2}$$

$$(COP)_{H.P.} + (COP)_{R.H.P.} = \frac{Q_1}{Q_1 - Q_2} + \frac{Q_2}{Q_1 - Q_2} = 1$$

$$[COP]_{H.P.} = 1 + (COP)_{R.H.P.}$$

(*) To show that Kelvin-Planck & Clausius statement as parallel statements of 2nd law of T.D.

[CASE I] [Violation of Clausius' statement]



violation of
Clausius' statement $\Rightarrow$

violation of Kelvin
Planck's statement

[CARNOT CYCLE] : (REVERSIBLE CYCLE)

num (41)

Interview

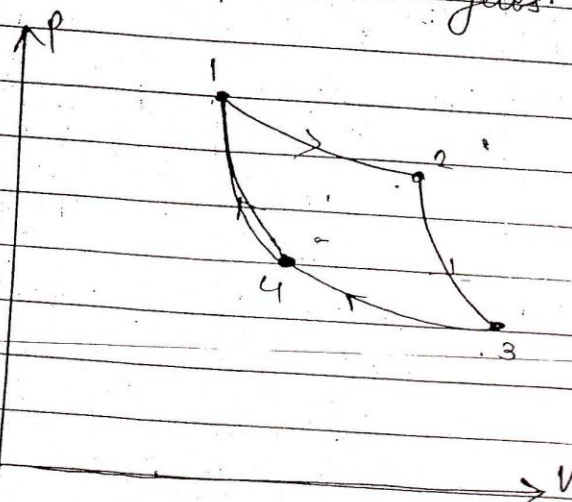
A cycle is said to be a reversible cycle only when each process in a cycle is a reversible process. Carnot cycle consists of two isothermal and two adiabatic processes. Isothermal process is a slow process and adiabatic process is a fast process and hence these two combinations in a cycle are very difficult to achieve. Therefore Carnot cycle is a theoretical cycle. And this cycle is used for comparing other practical cycles.

1-2 - Isothermal
heat addition (expⁿ)

2-3 - Adiabatic expansion

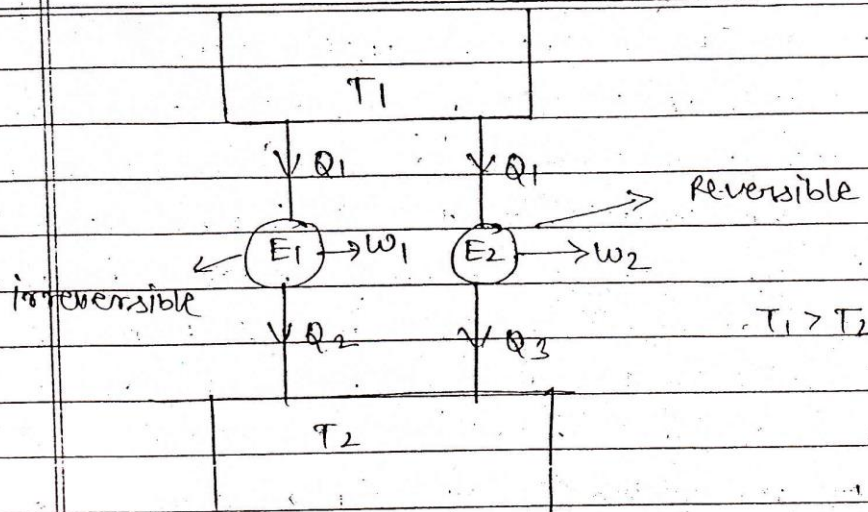
3-4 - Isothermal heat rejection (compression)

4-1 - Adiabatic compression



[CARNOT'S THEOREM] $\Rightarrow$ For various engines operating b/w same temperature limits no engine has efficiency greater than reversible cycle or Carnot cycle efficiency.

* let us consider two engines E_1 and E_2 operating b/w same temperature limits T_1 and T_2 .



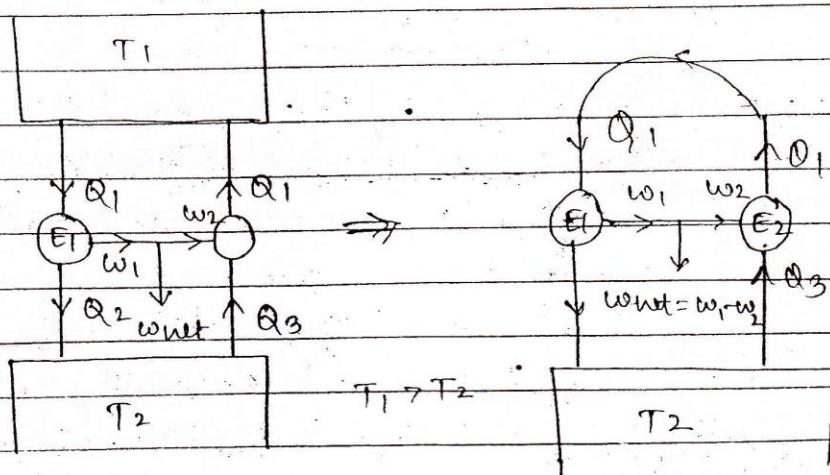
$$\eta_{\text{irrev}} = \frac{w_1}{Q_1} \quad \text{and} \quad \eta_{\text{rev}} = \frac{w_2}{Q_1}$$

Let's assume, $\eta_{\text{irrev}} \geq \eta_{\text{rev}}$.

$$\text{i.e.} - \frac{w_1}{Q_1} > \frac{w_2}{Q_1}$$

$$\Rightarrow w_1 > w_2$$

* As E_2 is a reversible engine let's reverse the direction.



$$w_{\text{net}} = w_1 - w_2$$

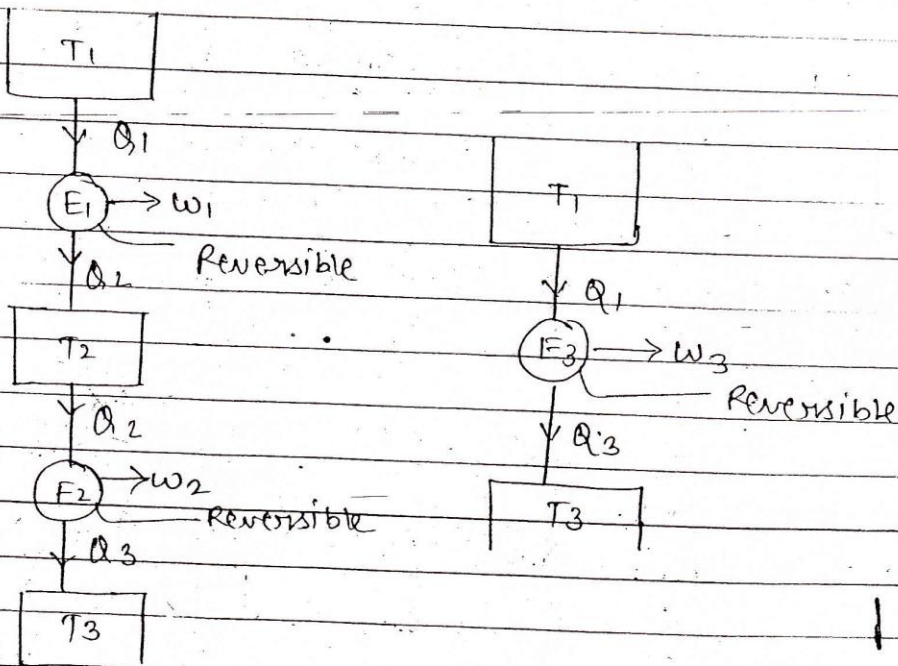
This system is producing work & exchanging heat with single reservoir.

* And this is the violation of Kelvin-Planck statement. Hence our assumption was wrong. (42)
 $\therefore \eta_{\text{irrev.}}$ can not be greater than $\eta_{\text{rev.}}$ operating b/w same temp limits.

 * Important points with respect to reversible cycle -

- (1) All reversible cycles operating b/w same temperature limits have same efficiency.
- (2) Efficiency of a reversible cycle is independent of working fluid.
- (3) Reversible cycle effo depends only on temperature limits.

<Thermodynamic temperature scale>



$$\eta_1 = 1 - \frac{Q_2}{Q_1} \Rightarrow \eta_1 = f_1(T_1, T_2) \rightarrow \text{Reversible cycle}$$

$$\eta_{\text{rev}} = 1 - \frac{Q_2}{Q_1} = f_1(T_1, T_2)$$

$$1 - f_1(T_1, T_2) = Q_2/Q_1$$

$$\frac{Q_1}{Q_2} = \frac{1}{[1 - f_1(T_1, T_2)]}$$

$$\Rightarrow \frac{Q_1}{Q_2} = \phi_1(T_1, T_2)$$

$$\text{ii) } \frac{Q_2}{Q_3} = \phi_2(T_2, T_3)$$

$$\frac{Q_1}{Q_3} = \phi_3(T_1, T_3)$$

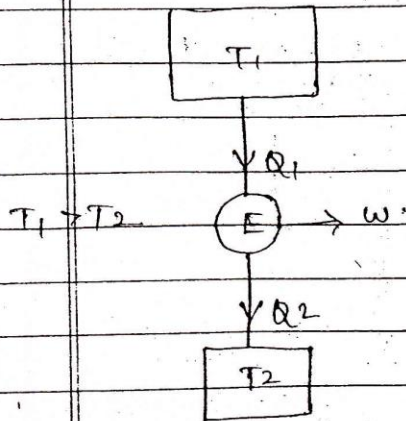
$$\Rightarrow \frac{Q_1}{Q_2} = \frac{Q_1/Q_3}{Q_2/Q_3} = \frac{\phi_3(T_1, T_3)}{\phi_2(T_2, T_3)} = \frac{\psi_1(T_1)}{\psi_2(T_2)}$$

$$\left\{ \frac{Q_1}{Q_2} = \frac{\psi_1(T_1)}{\psi_2(T_2)} \right\}_2$$

from the results of kelvin experiments

$$\left[\frac{Q_1}{Q_2} = \frac{T_1}{T_2} \right] \rightarrow \text{valid only for reversible cycle.}$$

[Efficiency of a reversible cycle]



$$\left\{ \eta = 1 - \frac{Q_2}{Q_1} \right\} \text{ for any cycle rev/irrev.}$$

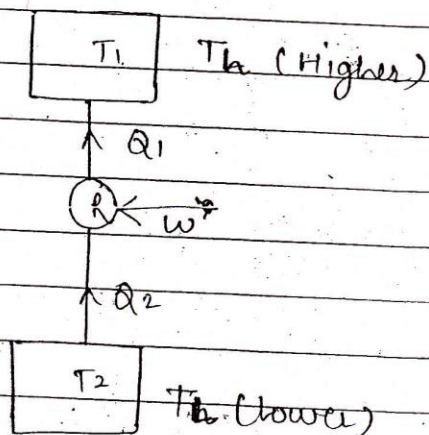
If the engine is reversible

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

$$\left\{ \eta_{\text{rev}} = 1 - \frac{T_2}{T_1} \right\}$$

or $\eta_{rev} = 1 - \frac{T_L}{T_H} \Rightarrow \eta_{max} = 1 - \frac{T_L}{T_H}$ (43)

[C.O.P. OF A REVERSIBLE REFRIGERATOR]



$$[COP]_R = \frac{Q_2}{Q_1 - Q_2} \rightarrow \text{for any ref. (irrev/rev)}$$

$$[COP]_R = \frac{1}{\frac{Q_1}{Q_2} - 1}$$

if cycle is reversible then

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

$$[COP_R]_{Rev} = \frac{1}{\frac{T_1}{T_2} - 1} = \frac{T_2}{T_1 - T_2}$$

$$[COP_R]_{Rev} = \frac{T_L}{T_H - T_L}$$

$$\left\{ [COP_R]_{max} = \frac{T_L}{T_H - T_L} \right\}$$

NOTE $\rightarrow$ cop of a reversible heat pump.

$$\frac{T_H}{T_H - T_L}$$

$$[(COP)_{H.P}]_{Rev} = \frac{T_H}{T_H - T_L}$$

*[CLAUSIUS INEQUALITY] * $\rightarrow$ [for a cycle]

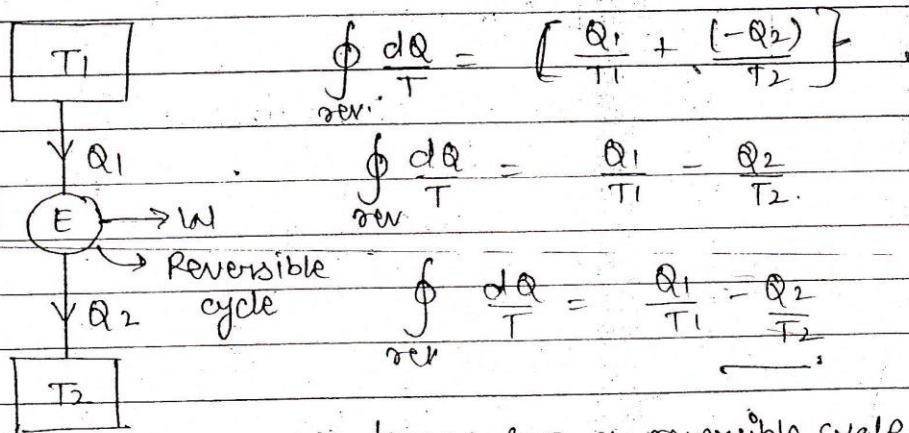
The cyclic integral of $\frac{dQ}{T}$ is less than or equal to zero.

i.e. $\oint \frac{dQ}{T} \leq 0$

if $\oint \frac{dQ}{T} < 0$ [cycle is irreversible]

if $\oint \frac{dQ}{T} = 0$ [" " Reversible].

[CASE 1] * [REVERSIBLE CYCLE] *



we know for a reversible cycle

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2} \Rightarrow \frac{Q_1}{T_1} = \frac{Q_2}{T_2}$$

$$\Rightarrow \oint_{\text{rev}} \frac{dQ}{T} = \frac{Q_1}{T_1} - \frac{Q_1}{T_1} = 0$$

hence

$$\oint_{\text{rev}} \frac{dQ}{T} = 0$$

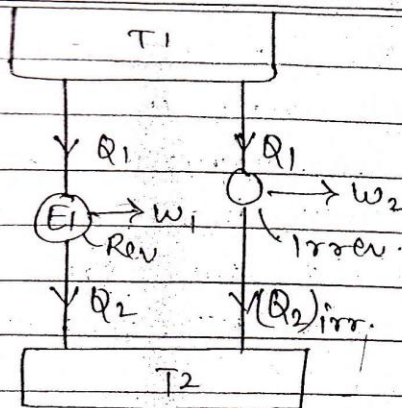
Proved -

[CASE (2) : IRREVERSIBLE CYCLE]

$$\left(\oint \frac{dQ}{T} \right)_{\text{irrev}}$$

$$= \frac{Q_1}{T_1} + \left(- \frac{Q_2 \text{ irrev}}{T_2} \right)$$

$$= \frac{Q_1}{T_1} - \frac{Q_2 \text{ irrev}}{T_2}$$



We know for
reversible cycle

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2} \quad (\text{for } E_1)$$

but $\left(\frac{Q_1}{T_1} \right)_{\text{rev}} = \left(\frac{Q_1}{T_1} \right)_{\text{irrev}} \rightarrow \text{here}$

$$\Rightarrow \left(\oint \frac{dQ}{T} \right)_{\text{irrev}} = - \frac{Q_2}{T_2} - \frac{Q_2 \text{ irrev}}{T_2}$$

$$\left(\oint \frac{dQ}{T} \right)_{\text{irrev}} = \frac{Q_2 - Q_2 \text{ irrev}}{T_2} \quad \text{--- (1)}$$

We know $\eta_1 = \eta_{\text{rev}} = \frac{W_1}{Q_1}$

$$\eta_2 = \eta_{\text{irrev}} = \frac{W_2}{Q_1}$$

is $\eta_{\text{rev}} > \eta_{\text{irrev}} \Rightarrow \frac{W_1}{Q_1} > \frac{W_2}{Q_1}$

$$W_1 > W_2$$

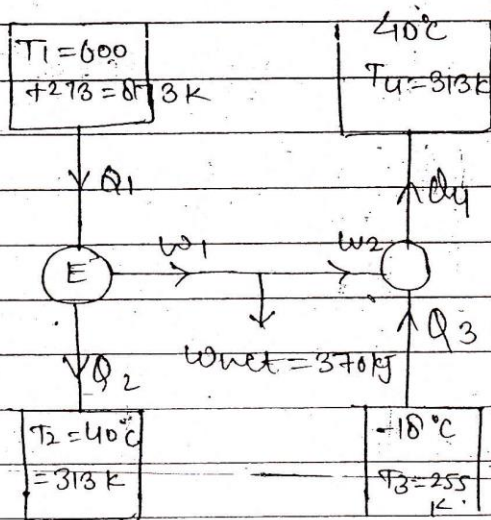
$$\Rightarrow Q_2 \text{ irrev} > Q_2$$

from (1) $\left. \oint \frac{dQ}{T} \right|_{\text{irrev}} = \frac{Q_2 - Q_2 \text{ irrev}}{T} < 0$

$$\Rightarrow \left[\left(\oint \frac{dQ}{T} \right)_{\text{irrev}} < 0 \right]$$

Question) A rev. heat engine operates b/w 600°C and 40°C . This engine drives a reversible refrigerator operating b/w 40°C and -18°C . Still there is a net work output of 370 kJ and the heat received by the engine is 2100 kJ . Determine the cooling effects

Solution)



$$(\text{COP})_R = \frac{Q_3}{w_2}$$

but ratio is reversible

$$(\text{COP})_R = \frac{Q_3}{w_2} = \frac{T_L}{T_H - T_L}$$

$$\frac{Q_3}{w_2} = \frac{255}{313 - 255} = 4.3965$$

$$\{Q_3 = 4.3965 w_2\}$$

$$w_1 - w_2 = 370$$

$$\text{but } w_1 = Q_1 - Q_2$$

$$\text{we know } \frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

$$Q_2 = \frac{Q_1 T_2}{T_1} = \frac{2100 \times 313}{873}$$

248 (45)

$$Q_2 = 752.9 \text{ KJ}$$

$$\text{So } W_1 = Q_1 - Q_2 = 2100 - 752.9$$

$$\{W_1 = 1347.1 \text{ KJ}\}$$

$$W_1 - W_2 = 370$$

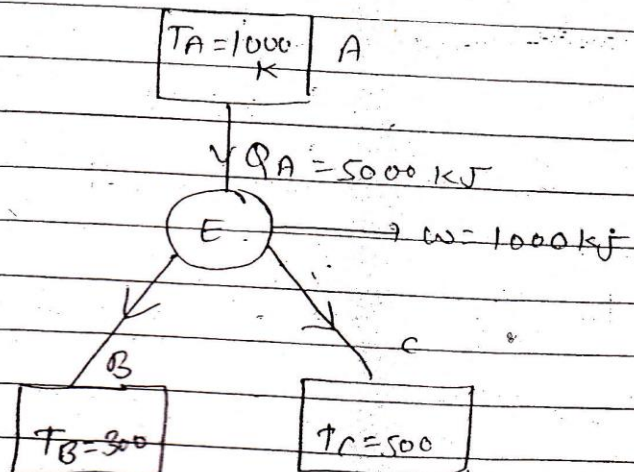
$$W_2 = (1347.1 - 370) \text{ KJ}$$

$$\{W_2 = 977.1 \text{ KJ}\}$$

$$Q_3 = 4.3965 W_2 = 4.3965 \times 977.1$$

$$Q_3 = 4295.34 \text{ KJ} \quad \underline{\text{Ans.}}$$

Question Fig. shows a reversible cycle during which it exchanges heat with three reservoirs and develops 1000 KJ of work. Find the magnitude of Q_A & dirⁿ of Q_B & Q_C .



→ for a reversible cycle

$$\oint_{\text{rev}} \frac{dQ}{T} = 0$$

$$\frac{5000}{1000} + \left(\frac{-Q_B}{300} \right) + \left(\frac{-Q_C}{500} \right) = 0$$

$$5 = \frac{Q_B}{300} + \frac{Q_C}{500} \quad \text{--- (1)}$$

And $5000 = W + Q_B + Q_C$

$$5000 = 1000 + Q_B + Q_C$$

$$Q_B + Q_C = 4000 \quad \text{--- (2)}$$

from (1) and (2)

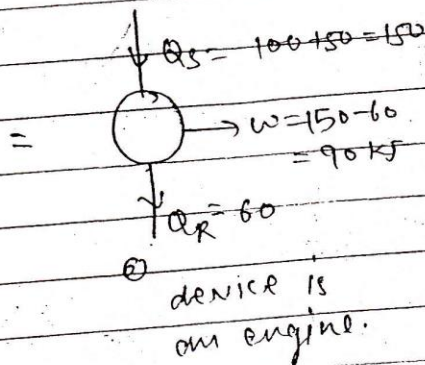
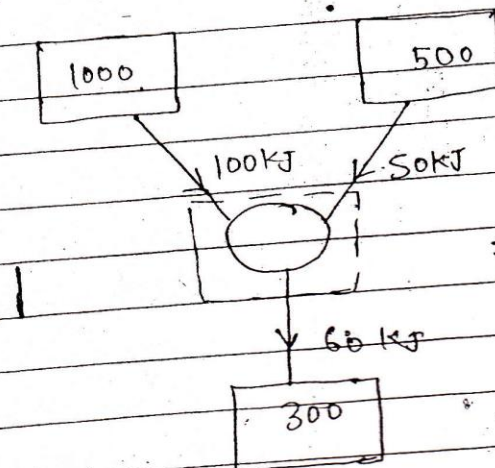
$$Q_B = -2250 \text{ KJ}$$

$$Q_C = 6250 \text{ KJ}$$

Hence $Q = 2250 \text{ KJ}$ (from B to E) } Ans
 $Q_C = 6250 \text{ KJ}$ (from E to C)

[Workbook - Ch. (4)]

Q.1

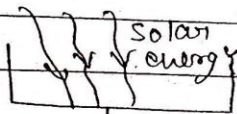


$$\oint \frac{dQ}{T} = \frac{1000}{1000} - \frac{100 + 50}{500} - \frac{60}{300}$$

$$= \frac{1}{10} + \frac{1}{10} - \frac{1}{5}$$

$$= \frac{2}{10} - \frac{1}{5} = 0 \quad (\text{Reversible})$$

Q.2



$Q_1 = 0.5 \frac{\text{KW}}{\text{m}^2} = 0.5 \times A \text{ KW}$

→ area

$W = 1 \text{ KW}$

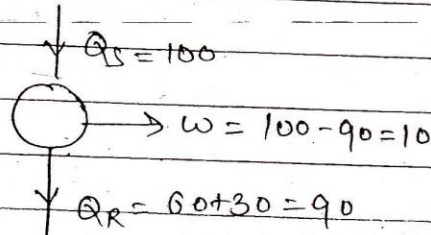
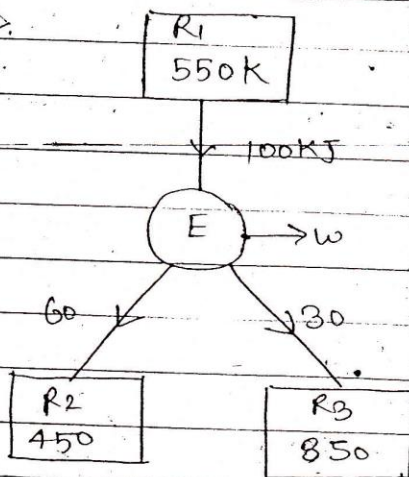
$\eta = \frac{W}{Q_1}$

$\eta = 0.2$

$0.2 = \frac{1}{0.5 \times A}$

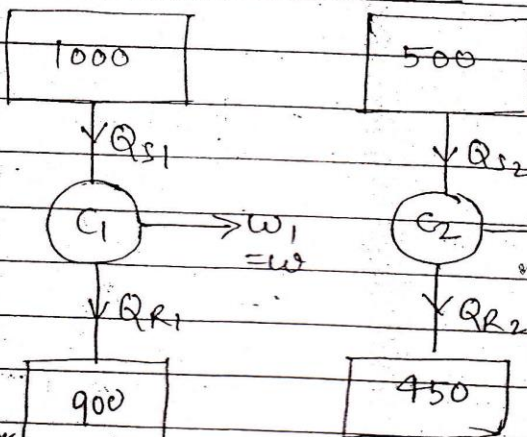
$A = \frac{1}{0.2 \times 0.5} = \frac{100}{10} = 10 \text{ m}^2$

Q.3



$\eta = \frac{10}{100} = 0.1$

Q.4



$\eta_1 = 1 - \frac{T_L}{T_H}$

$= 1 - \frac{900}{1000}$

$\eta = 0.1$

$\eta_2 = 1 - \frac{T_L}{T_H}$

$= 1 - \frac{450}{500}$

$\eta_2 = 0.1$

$$\eta_1 = \eta_2$$

$$\frac{w_1}{Q_{S1}} = \frac{w_2}{Q_{S2}} \Rightarrow \frac{w}{Q_{S1}} = \frac{w}{Q_{S2}}$$

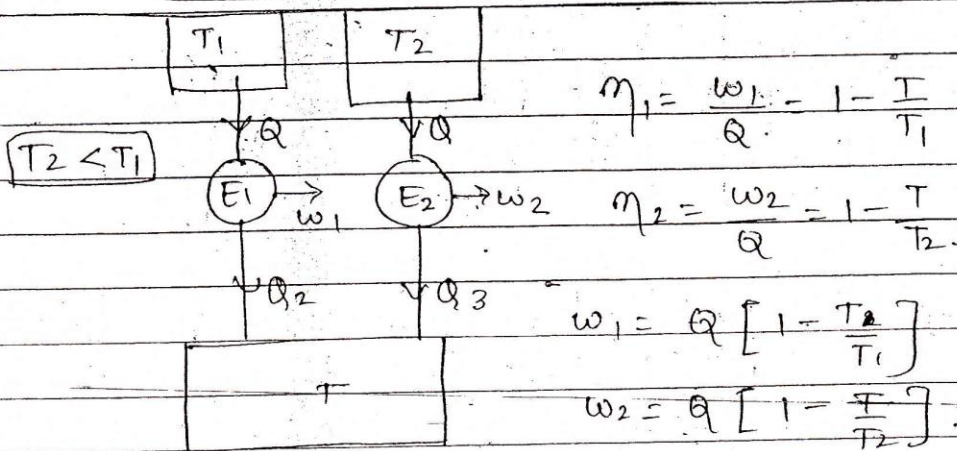
$$Q_{S1} = Q_{S2}$$

$$w_1 = w_2$$

$$Q_{S1} - Q_{R1} = Q_{S2} - Q_{R2}$$

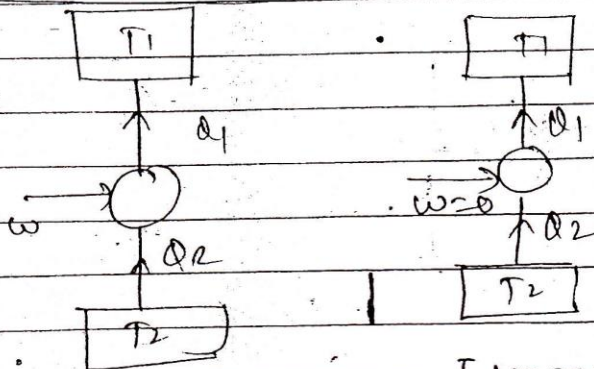
$$Q_{R1} = Q_{R2}$$

Q.10

given $T_2 < T_1$

$$\Rightarrow w_2 < w_1$$

(11)



[Impossible]

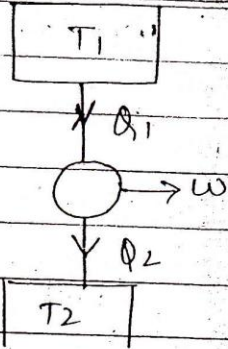
Possible

$$\text{COP} = \frac{Q_2}{w}$$

$$\text{COP} = \frac{Q_2}{w} = \frac{Q_2}{0} = \infty$$

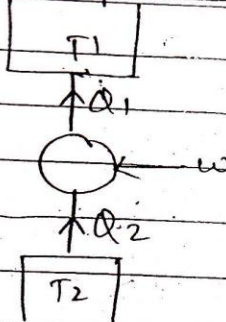
COP = ∞ (impossible)

(12)



$$\eta_{\text{COP}} = \frac{W}{Q_1}$$

Reverse



$$(COP)_{H.P} = \frac{Q_1}{W} = \frac{1}{\eta_{\text{max}}}$$

$$(COP)_R = (COP)_{H.P} - 1$$

$$(COP)_{\text{Ref}} = \frac{1}{\eta_{\text{max}}} - 1$$

Q.13 $T_H = 627 + 273 = 900\text{K}$

$T_L = 27 + 273 = 300\text{K}$

$$\eta_{\text{rev}} = \eta_{\text{max}} = 1 - \frac{T_L}{T_H} = 1 - \frac{300}{900} = 66.7\%$$

Inventor engine.

output = 50 kW

$CV = 75000 \text{ kJ/kg}$

$m_f = 3 \text{ kg/hr}$

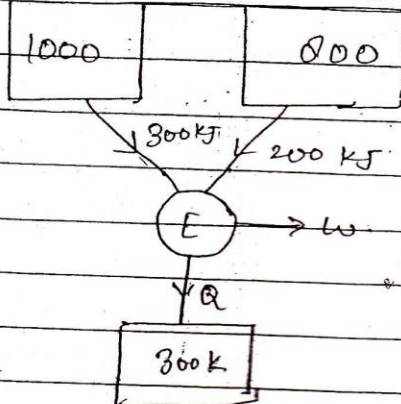
Input = $\frac{75000 \times 3}{3600} \frac{\text{kJ}}{\text{s}}$

I/P = $62.5 \text{ kJ/s} = 62.5 \text{ kW}$

$$\eta_{\text{inventor}} = \frac{O/P}{I/P} = \frac{50}{62.5} = 0.8 = 80\%$$

$\eta_{\text{rev}} < \eta_{\text{inventor}}$ (Impossible).

14



Engine is reversible

$$\oint \frac{dQ}{T} = 0$$

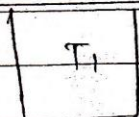
$$\frac{300}{1000} + \frac{200}{800} - \frac{Q}{300} = 0$$

$$\frac{3}{10} + \frac{1}{4} - \frac{Q}{300} = 0$$

$$Q = 165 \text{ kJ}$$

$$W = 300 + 200 - 165 = 335 \text{ kJ}$$

Q.16



$$\eta_1 = 1 - \frac{T_2}{T_1}$$

$$w_1 = Q_1 - Q_2$$

$$w_2 = Q_2 - Q_3$$

$$\eta_2 = 1 - \frac{T_3}{T_2}$$

$$w_1 = w_2$$

$$\eta_1 = \eta_2$$

$$Q_1 - Q_2 = Q_2 - Q_3$$

$$1 - \frac{T_2}{T_1} = 1 - \frac{T_3}{T_2}$$

$$w_1 = Q_1 - Q_2$$

$$= Q_2 \left(\frac{Q_1}{Q_2} - 1 \right)$$

$$\frac{T_2}{T_1} = \frac{T_3}{T_2}$$

$$= Q_2 \left(\frac{T_1}{T_2} - 1 \right)$$

$$T_2 = \sqrt{T_1 T_3}$$

$$w_2 = Q_2 - Q_3 = Q_2 \left(1 - \frac{Q_3}{Q_2} \right) = Q_2 \left(1 - \frac{T_3}{T_2} \right)$$

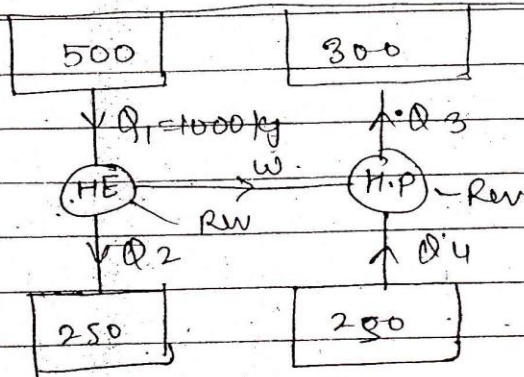
$$w_1 = w_2$$

$$Q_2 \left(\frac{T_1}{T_2} - 1 \right) = Q_2 \left(1 - \frac{T_3}{T_2} \right)$$

$$\frac{T_1}{T_2} = \frac{T_3}{T_2} + 2$$

$$T_1 = T_3 + 2T_2 \Rightarrow T_2 = \frac{T_1 + T_3}{2}$$

10



$$(COP)_{HP} = \frac{T_h}{T_h - T_c}$$

$$COP = \frac{300}{300 - 250}$$

$$(COP)_{HP} = 6$$

$$(COP)_{HP} = \frac{Q_3}{w}$$

$$Q_3 = 6 \times w \quad \text{--- (1)}$$

$$w = Q_1 - Q_2$$

$$w = 1000 - Q_2$$

$$Q_3 = 6 \times 500 = 3000 \text{ kJ}$$

$$\frac{Q_1}{T_1} - \frac{Q_2}{T_2} = 0$$

$$Q_2 = \frac{Q_1 \times T_2}{T_1} = 500 \Rightarrow w = 1000 - 500 = 500$$

Eduwrte

CHAPTER-5

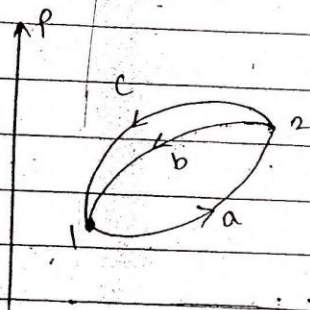
* ENTROPY *

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CASE I → REVERSIBLE CYCLE

26/48



we know

$$\oint_{\text{rev}} \frac{\delta Q}{T} = 0$$

for cycle 1-a-2-b-1

$$\oint_{\text{rev}} \frac{\delta Q}{T} = \left(\frac{\delta Q}{T} \right)_{1a2} + \left(\frac{\delta Q}{T} \right)_{2b1} = 0$$

for cycle 1-a-2-c-1

$$\oint_{\text{rev}} \frac{\delta Q}{T} = \left(\frac{\delta Q}{T} \right)_{1a2} + \left(\frac{\delta Q}{T} \right)_{2c1} = 0$$

$$\text{Eq}^n (1) - \text{Eq}^n (2)$$

$$\left(\frac{\delta Q}{T} \right)_{2b1} - \left(\frac{\delta Q}{T} \right)_{2c1} = 0$$

$$\left(\frac{\delta Q}{T} \right)_{2b1} = \left(\frac{\delta Q}{T} \right)_{2c1}$$

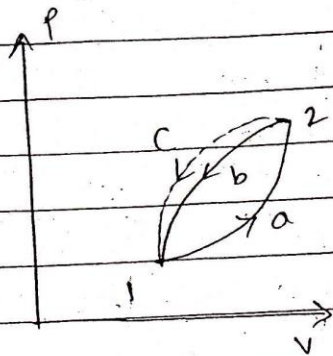
* $\left(\frac{\delta Q}{T} \right)_{\text{rev}}$ along paths b & c are same. Therefore $\left(\frac{\delta Q}{T} \right)_{\text{rev}}$ is independent of path and it depends only on end points.

Therefore $\left(\frac{\delta Q}{T} \right)_{\text{rev}}$ must be a property. And this property is known as ENTROPY

$$\Rightarrow \left(\frac{\delta Q}{T} \right)_{\text{rev}, b} = \left(\frac{\delta Q}{T} \right)_{\text{rev}, c} = ds$$

$$ds = \left(\frac{\delta Q}{T} \right)_{\text{REVERSIBLE}}$$

CASE (2) [IRREVERSIBLE CYCLE]



1-a-2-b-1 → Reversible cycle

$$\left(\frac{dQ}{T}\right)_{1a2} + \left(\frac{dQ}{T}\right)_{2b1} = 0$$

$$\left(\frac{dQ}{T}\right)_{1a2} = - \left(\frac{dQ}{T}\right)_{2b1} \quad \text{--- (1)}$$

Now considering

1-a-2-c-1 (Irreversible cycle)

$$\oint_{\text{irrev}} \frac{dQ}{T} < 0$$

$$\left(\frac{dQ}{T}\right)_{1a2} + \left(\frac{dQ}{T}\right)_{2c1} < 0$$

$$\text{from (1)} \quad - \left(\frac{dQ}{T}\right)_{2b1} + \left(\frac{dQ}{T}\right)_{2c1} < 0$$

$$\text{OR} \quad \left(\frac{dQ}{T}\right)_{2c1}^{\text{irrev}} < \left(\frac{dQ}{T}\right)_{2b1}^{\text{rev}}$$

$$\left(\frac{dQ}{T}\right)_{\text{irrev}} < \left(\frac{dQ}{T}\right)_{\text{rev}}$$

$$\left(\frac{dQ}{T}\right)_{\text{irrev}} < ds$$

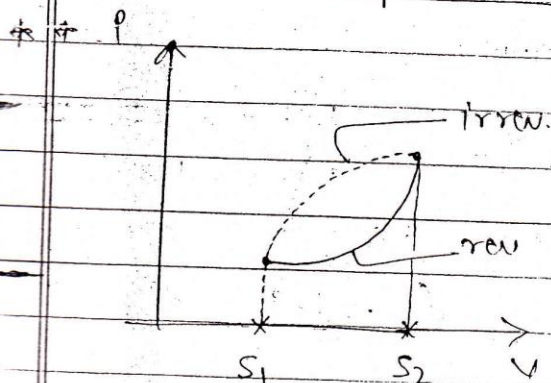
$$\Rightarrow \boxed{ds > \left(\frac{dQ}{T}\right)_{\text{irrev.}}}$$

we know $ds = \left(\frac{\delta Q}{T}\right)_{\text{rev}}$ from case ① (49)

$ds > \left(\frac{\delta Q}{T}\right)_{\text{irrev}}$ from case ②

Combining both the cases

$$ds \geq \left(\frac{\delta Q}{T}\right)$$



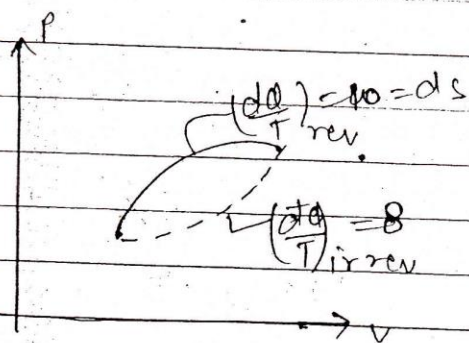
$$\Delta S_{\text{rev}} = S_2 - S_1$$

$$\Delta S_{\text{irrev}} = S_2 - S_1$$

because entropy is a property, doesn't depend upon the path.

if ends

$$\Delta S_{\text{rev}} = \Delta S_{\text{irrev}} \rightarrow \text{for two given points}$$



$$\left\{ ds > \left(\frac{\delta Q}{T}\right)_{\text{irrev}} \right\}$$

Imp

As entropy is a property, it depends only on end points and hence as long as the end points are same, the entropy change will be same irrespective of the process i.e. b/w any two given points, the

Entropy change for rev. & irrev. path will be same. In order to find out the entropy change for irreversible process it must be replaced by a reversible process b/w same end points.

* ENTROPY CHANGE OF SYSTEM IN REVERSIBLE PROCESS :-

* [CASE 1] : [Heat Addition]

we know $(ds) = \frac{dq}{T}$ [for reversible process]

if heat is added $dq > 0$

& $T > 0$ (always) in K

$$\Rightarrow ds > 0$$

$$S_2 - S_1 > 0$$

$$\boxed{S_2 > S_1}$$

* System entropy increases, when heat is added in a reversible process.

CASE (2) : [Heat rejection]

$$ds = \frac{dq}{T} \quad (\text{Reversible})$$

if heat is rejected $\Rightarrow \frac{ds}{T} \rightarrow -ve$

$$ds < 0$$

$$S_2 - S_1 < 0$$

$$\boxed{S_2 < S_1}$$

- System entropy decreases, in a reversible process when heat is rejected. 253 ⑥

[CASE - 3] : [Reversible adiabatic process]

$$ds = \frac{dq}{T}$$

is valid for (i) Reversible process ~~ET~~
and here we are having (ii) Adiabatic process

$$\Rightarrow ds = \frac{dq}{T}$$

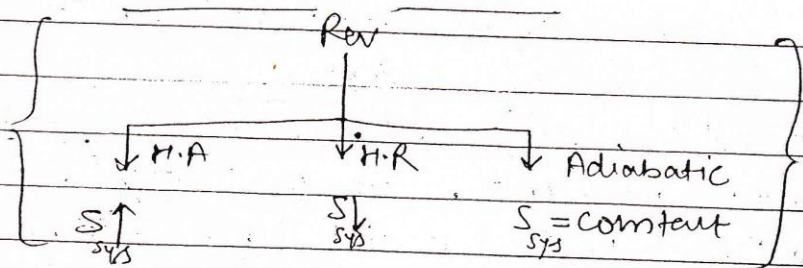
for adiabatic process $dq = 0$

$$ds = 0$$

$$s_2 - s_1 = 0$$

$$\Rightarrow * \boxed{s_2 = s_1} \rightarrow \text{Isentropic}$$

"A reversible adiabatic process is always an isentropic process."



*** In a reversible process system entropy can increase, decrease or can remain constant depending on heat transfer.

* ENTROPY CHANGE IN IRREVERSIBLE PROCESS : —

Suppose we have a system, losing heat.

$$\text{System} \rightarrow Q \quad (ds) > \left(\frac{dq}{T}\right)_{\text{irrev}}$$

(OR)

$$ds = \left(\frac{dq}{T}\right)_{\text{irrev}} + (ds)_{\text{gen}} \quad \rightarrow \text{+ve}$$

Let's take $\left(\frac{dq}{T}\right)_{\text{irrev}} = -3 \text{ unit}$

$$ds - \frac{dq}{T} < 0$$

and $(ds)_{\text{gen}} = +3 \text{ unit}$ let say

$$ds = -3 + 3 = 0$$

$$ds = 0$$

$$[S_2 = S_1] \rightarrow \text{Isentropic process}$$

although it is not reversible
and not adiabatic.

* Though a reversible adiabatic process is always isentropic. An isentropic process need not be reversible adiabatic.

ADIABATIC PROCESS ($\delta Q = 0$)

Reversible

$$ds = \frac{\delta Q}{T}$$

$$ds = \frac{0}{T} = 0$$

$$ds = 0$$

$$S = \text{constant}$$

(Isentropic)

Hence -

Irreversible

$$ds = \left(\frac{\delta Q}{T}\right)_{\text{irrev}} + \delta s_{\text{gen}}$$

$$ds = 0 + \delta s_{\text{gen}}$$

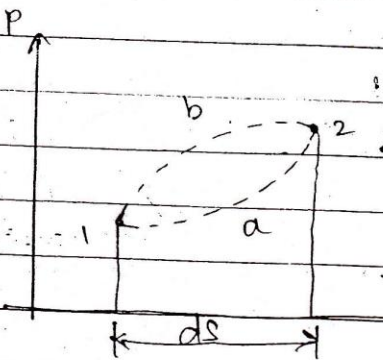
↓ +ve

$$ds > 0$$

$$\Rightarrow S_2 - S_1 > 0 \Rightarrow S \uparrow$$

(Non-isentropic)

An adiabatic process can be isentropic or non-isentropic. If the adiabatic process is isentropic it must be reversible.



$$ds = \left(\frac{\delta Q}{T}\right)_{\text{irrev}} + \delta s_{\text{gen}}$$

$$ds_a = \left(\frac{\delta Q}{T}\right)_{\text{irrev},a} + (\delta s_{\text{gen}})_a$$

$$ds_b = \left(\frac{\delta Q}{T}\right)_{\text{irrev},b} + (\delta s_{\text{gen}})_{\text{gen},b}$$

$$[ds_a = ds_b]$$

Clearly

as end points are same

Let a be more irreversible than b

$$\Rightarrow \boxed{(\delta s_{\text{gen}})_a > (\delta s_{\text{gen}})_b}$$

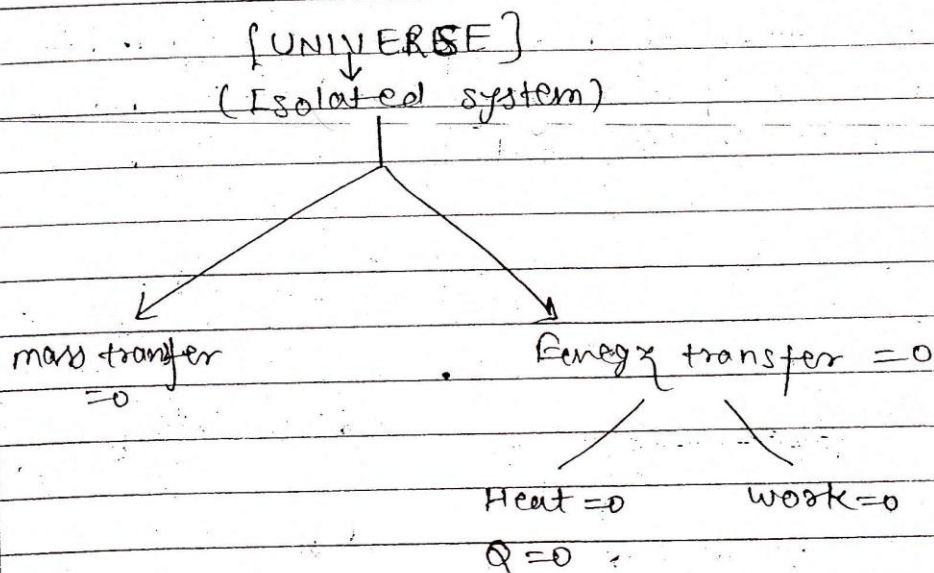
Though entropy is a property, entropy generation is not a property. It depends on the extent of irreversibilities i.e. greater the irreversibility, greater is the entropy generation.

* PHYSICAL MEANING OF ENTROPY *

Entropy is a measure of randomness of molecules. i.e. greater the disorderliness, greater is the entropy and lesser is the efficiency.

* Entropy is an extensive property.

* [ENTROPY CHANGE OF UNIVERSE] *



Hence for universe $Q = 0$

we know, $ds \geq \frac{dQ}{T}$

$$(ds)_{\text{universe}} \geq 0 \quad \checkmark \quad \text{****}$$

$\Rightarrow$

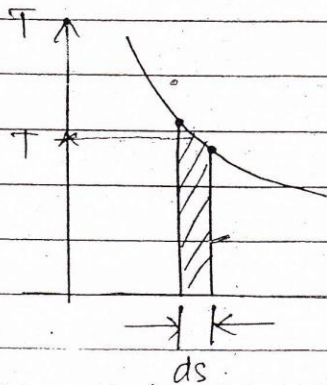
$$(ds)_{\text{system}} + (ds)_{\text{surrounding}} \geq 0$$

Eduwrte

7/0

According to 2nd law of T.D. all those processes are possible for which the change in entropy of universe is greater than or equal to zero. There can never be entropy decrease of universe. And this is known as the principle of increase of entropy.

* [T-S DIAGRAM] *



$$\text{Area} = T \cdot ds \quad \text{--- (1)}$$

we know

$$ds = \frac{dq}{T} \quad \text{[for reversible]}$$

$$dq = T ds \quad \text{[Reversible]} \quad \text{--- (2)}$$

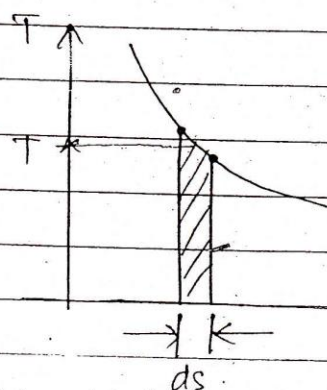
from (1) and (2)

$$dq = \text{Area under T-S diagram.}$$

4-1 → Adiabatic compression

According to 2nd law of T.D. all those processes are possible for which the change in entropy of universe is greater than or equal to zero. There can never be entropy decrease of universe. And this is known as the principle of increase of entropy. (2)

* [T-S DIAGRAM] *



$$\text{Area} = T \cdot ds \quad \text{--- (1)}$$

we know

$$ds = \frac{dQ}{T} \quad \text{[for reversible]}$$

$$dQ = T ds \quad \text{(Reversible)} \quad \text{--- (2)}$$

from (1) and (2)

$$dQ = \text{Area under } Tds \text{ diagram.}$$

Area under the curve when projected on entropy axis gives reversible heat transfer.

* Representation of Carnot cycle on T-s diagram
In a Carnot cycle

1-2 → Isothermal heat addition

2-3 → Adiabatic expn

3-4 → Isothermal heat rejection

4-1 → Adiabatic compression

we know

Reversible process

Q_{supply}

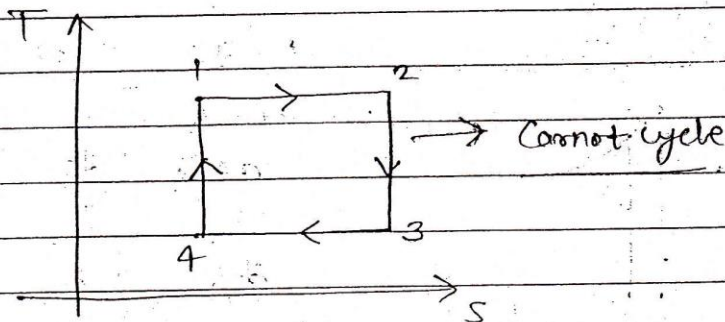
$S \uparrow$

Q_{reject}

$S \downarrow$

$Q = 0$ adiabatic

$S = \text{constant}$



[COMBINED I & II LAW OF THERMODYNAMICS]

I Law

$$dQ = dU + PdV \rightarrow \text{for reversible process}$$

$$\frac{dQ}{T} = dS \rightarrow \text{for reversible}$$

$$dQ = T \cdot dS \text{ --- 2nd law}$$

from ① and ②

$$Tds = dU + PdV$$

Valid for ANY PROCESS

There is no path function in the above equations, all are the properties. That is why this is valid of any process.

Considering 1st law

$$dQ = du + PdV$$

$$H = U + PV$$

$$dH = du + PdV + VdP$$

$$dH = dQ + VdP$$

but from 2nd law $Tds = dQ$

$$\Rightarrow dH = Tds + VdP$$

$$\checkmark \boxed{Tds = dH - VdP} \quad \text{Valid for ANY PROCESS becoz}$$

each quantity is a property

* REPRESENTATION OF CONSTANT VOLUME AND CONSTANT PRESSURE LINES ON T-S DIAGRAM.

we know

$$Tds = du + PdV \rightarrow \text{specific quantities at constant volume}$$

$$PdV = 0$$

$$Tds = du$$

$$\text{for ideal gas } du = C_v dT$$

$$\Rightarrow Tds = C_v dT$$

$$\boxed{\frac{dT}{ds} = \frac{T}{C_v}}$$

OR

$$\boxed{\text{slope of constant volume curve on T-s diagram} = \frac{T}{C_v}}$$

Now considering

$$Tds = dh - vdp$$

if pressure is constant

$$vdp = 0$$

$$Tds = dh$$

for ideal gas $dh = c_p dT$

$$Tds = c_p dT$$

$$\boxed{\frac{dT}{ds} = \frac{T}{c_p}}$$



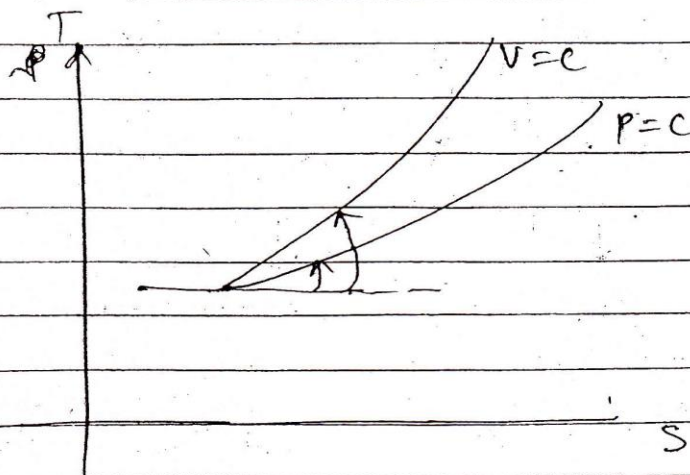
Slope of constant pressure curve
on Tds diagram = $\frac{T}{c_p}$

we know $c_p > c_v$

$$\frac{T}{c_p} < \frac{T}{c_v}$$

ie

$$\boxed{\left(\frac{dT}{ds}\right)_p < \left(\frac{dT}{ds}\right)_v}$$



[ENTROPY CHANGE FOR AN IDEAL GAS]

(54)

$$TdS = dU + PdV$$

for ideal gas

$$PV = mRT$$

$$\& dU = mc_v dT$$

$$ds = \frac{dU}{T} + \frac{P}{T} dV$$

$$ds = \frac{mc_v dT}{T} + \frac{mR dV}{V}$$

$$\int_{S_1}^{S_2} ds = \int_{T_1}^{T_2} \frac{mc_v dT}{T} + \int_{V_1}^{V_2} \frac{mR dV}{V}$$

$$\Rightarrow (S_2 - S_1) = mc_v \ln\left(\frac{T_2}{T_1}\right) + mR \ln\left(\frac{V_2}{V_1}\right)$$

$$TdS = dH - VdP$$

for ideal gas

$$PV = mRT$$

$$\& dH = mc_p dT$$

$$ds = \frac{dH}{T} - \frac{V}{T} dP$$

$$ds = \frac{mc_p dT}{T} - \frac{mR dP}{P}$$

$$\int_{S_1}^{S_2} ds = \int_{T_1}^{T_2} \frac{mc_p dT}{T} - \int_{P_1}^{P_2} \frac{mR dP}{P}$$

$$\Rightarrow (S_2 - S_1) = mc_p \ln\left(\frac{T_2}{T_1}\right) - mR \ln\left(\frac{P_2}{P_1}\right)$$

hence for an ideal gas

$$S_2 - S_1 = mc_v \ln\left(\frac{T_2}{T_1}\right) + mR \ln\left(\frac{V_2}{V_1}\right)$$

when T & V are given

$$S_2 - S_1 = mc_p \ln\left(\frac{T_2}{T_1}\right) - mR \ln\left(\frac{P_2}{P_1}\right)$$

when T & P are known

or

$$\left\{ \begin{array}{l} S_2 - S_1 = c_v \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{V_2}{V_1}\right) \\ S_2 - S_1 = c_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right) \end{array} \right\}$$

valid for
ideal gas
only

question) Show that for an ideal gas

$$s_2 - s_1 = C_v \ln \left(\frac{P_2}{P_1} \right) + C_p \ln \left(\frac{V_2}{V_1} \right).$$

Soln

$$s_2 - s_1 = C_v \ln \left(\frac{T_2}{T_1} \right) + R \ln \left(\frac{V_2}{V_1} \right) \quad \text{--- (1)}$$

$$s_2 - s_1 = C_p \ln \left(\frac{P_2}{P_1} \right) - R \ln \left(\frac{P_2}{P_1} \right) \quad \text{--- (2)}$$

from (1)

$$\begin{aligned} s_2 - s_1 &= C_v \ln \left(\frac{T_2}{T_1} \right) + (C_p - C_v) \ln \left(\frac{V_2}{V_1} \right) \\ &= C_v \left[\ln \left(\frac{T_2}{T_1} \right) + \ln \left(\frac{V_2}{V_1} \right) \right] + C_p \ln \left(\frac{V_2}{V_1} \right) \\ &= C_v \left[\ln \left(\frac{T_2}{T_1} \times \frac{V_2}{V_1} \right) \right] + C_p \ln \left(\frac{V_2}{V_1} \right) \end{aligned}$$

$$s_2 - s_1 = C_v \ln \left(\frac{T_2 V_2}{T_1 V_1} \right) + C_p \ln \left(\frac{V_2}{V_1} \right)$$

for ideal gas

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$\frac{T_2}{T_1} \times \frac{V_2}{V_1} = \frac{P_2}{P_1}$$

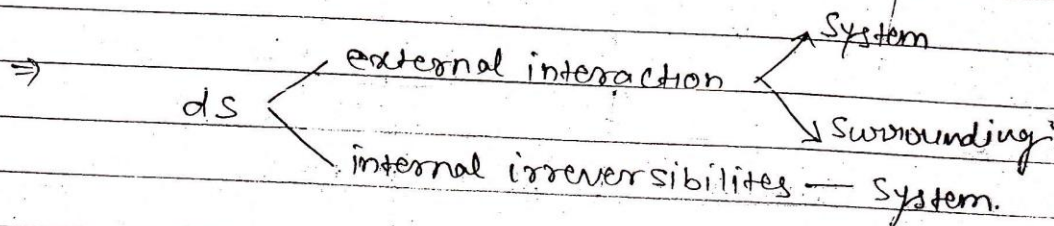
$$s_2 - s_1 = C_v \ln \left(\frac{P_2}{P_1} \right) + C_p \ln \left(\frac{V_2}{V_1} \right)$$

Proved

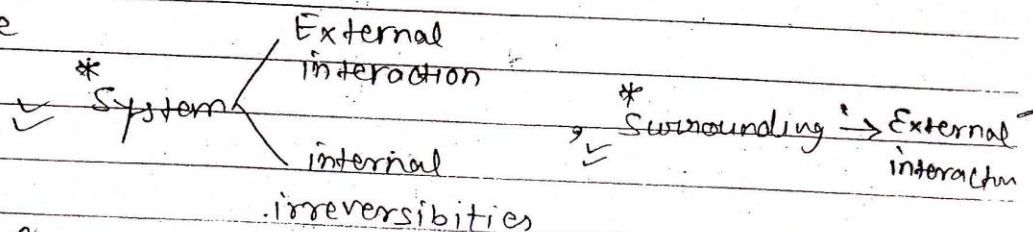
258 (5)

258 (5)

↓
Internal
irreversibilities



hence



~~V.V. GMP~~

In an adiabatic process as there is no interaction of system with surroundings in the form of heat transfer ($\frac{dQ}{T} = 0$) therefore there is no entropy change for surroundings in an adiabatic process.

* The system entropy can change in an is adiabatic process due to internal irreversibilities only

~~Imk.~~

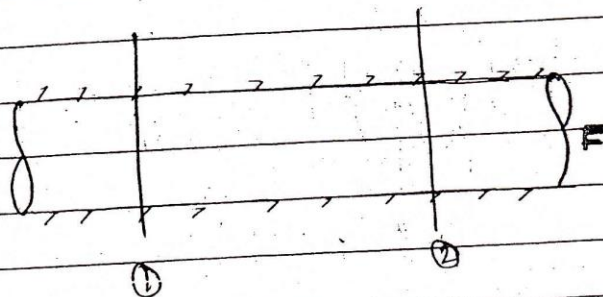
Question → Air is flowing steadily in an insulated pipe. The pressure and temp. at two section ① & ② are given in the table. Establish the direction of flow of air in pipe treating air as ideal gas. Take $C_p = 1.005 \text{ kJ/kg} \cdot \text{K}$

~~$R = 0.207 \text{ kJ/kg} \cdot \text{K}$~~

Eduvršte

	Section ①	Section ②
Pressure	130 kPa	100 kPa
Temperature	50 °C	13 °C

Solution →



Let's flow is from ① to ②

we know

$$(ds)_{\text{universe}} \geq 0$$

$$(ds)_{\text{system}} + (ds)_{\text{surroundings}} \geq 0$$

as pipe is insulating $\Rightarrow$ external heat interaction = 0

$$\text{i.e. } (ds)_{\text{surroundings}} = 0$$

$$\Rightarrow (ds)_{\text{system}} \geq 0$$

$$(ds)_{\text{system}} = (S_2 - S_1) = C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

$$S_2 - S_1 = 1.005 \ln \left(\frac{13+273}{50+273} \right) - 0.287 \ln \left(\frac{100}{130} \right)$$

$$S_2 - S_1 = -0.0469 \text{ kJ/kg-K}$$

$$\text{i.e. } (ds)_{\text{system}} < 0$$

Hence flow is not possible from ① to ②

$\Rightarrow$ Direction of flow is from ② to ①. $\star$

* ENTROPY CHANGE FOR SOLIDS/LIQUIDS.

259 (32)

we know, for any substance

$$Tds = du + PdV$$

but for solids/liquids $dV = 0$ [Volume is constant]

$$\Rightarrow Tds = du$$

for any substance $du = mc dT$

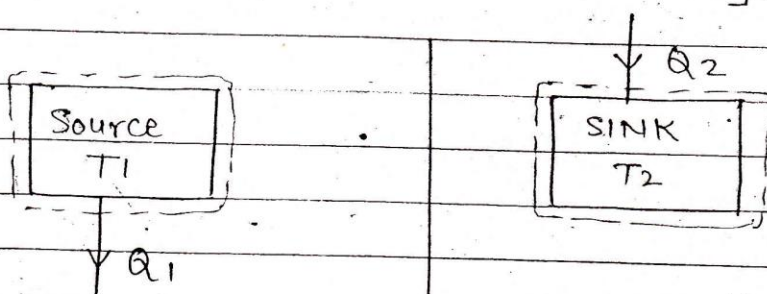
$$\Rightarrow Tds = mc dT$$

$$ds = mc \frac{dT}{T}$$

$$\int_{S_i}^{S_f} = mc \int_{T_i}^{T_f} \frac{dT}{T}$$

$$S_f - S_i = mc \ln \left(\frac{T_f}{T_i} \right)$$

* [ENTROPY CHANGE FOR A RESERVOIR] *



$$(ds)_{\text{source}} = -\frac{Q_1}{T_1}$$

$$(ds)_{\text{sink}} = \frac{Q_2}{T_2}$$

** Temperatures are in Kelvin

[Workbook] [Chapter - 5]

Q.2

Given $T = c, \Rightarrow dT = 0$.Incompressible fluid $[v = \text{constant}]$.

$$Tds = du + Pdv$$

$$Tds = du + 0 = du$$

$$\text{we know } du = mc(dT) \rightarrow 0$$

$$du = 0$$

$$[u = \text{constant}]$$

$$Tds = 0$$

$$ds = 0 \Rightarrow [s = \text{constant}]$$

Q.3

$$P_1 = 100 \quad T_1 = 300$$

$$P_2 = 200 \quad T_2 = 500$$

for ideal gas

$$ds = C_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right)$$

$$= \frac{5 \times 20.7}{2} \ln\left(\frac{5}{3}\right) - 20.7 \ln(2)$$

$$= 5 \left[2.07 \times 20.7 \ln\left(\frac{5}{3}\right) - 20.7 \ln(2) \right]$$

$$C_p - C_v = R$$

$$C_p = R + C_v = R + \frac{5}{2}R = \frac{7}{2}R$$

$$= 10 \left[3.5 \times 20.7 \ln\left(\frac{5}{3}\right) - 20.7 \ln(2) \right]$$

$$= 10 \quad \checkmark$$

$$= 3.1419 \text{ kJ/K}$$

Q.4

Second

$$ds = 0$$

$$[dT = T_2 - T_1]$$

$$Tds = du + PdV$$

$$du = 0$$

$$m \text{ ed } dT = 0$$

$$dT = 0 \quad \checkmark$$

Q.6 option (a) ✓

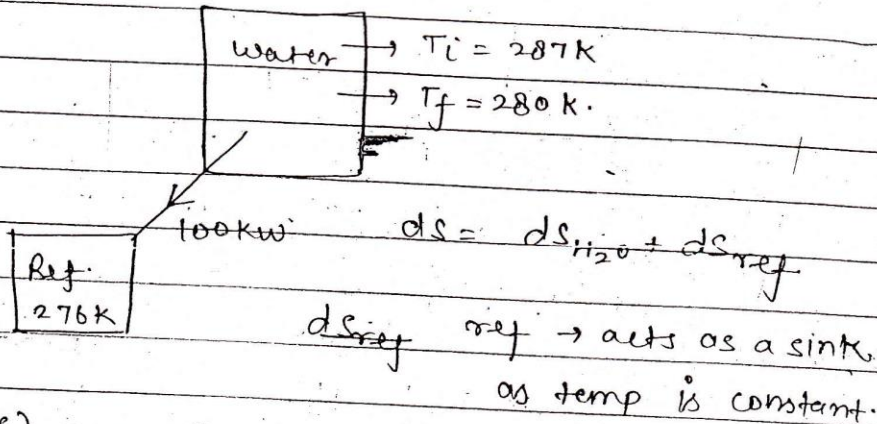
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option (b) $H = U + PV$

$H = f(t) + mRT$ ✓

(c) X.

Q.7 →



$$(ds)_{ref} = \frac{Q}{T} = \frac{100 \text{ kW}}{276 \text{ K}}$$

$$(ds)_{H_2O} = mc \ln \left(\frac{T_f}{T_i} \right)$$

we don't know (m)

$$\Rightarrow Q = mc \Delta T$$

$$100 = mc (287 - 280)$$

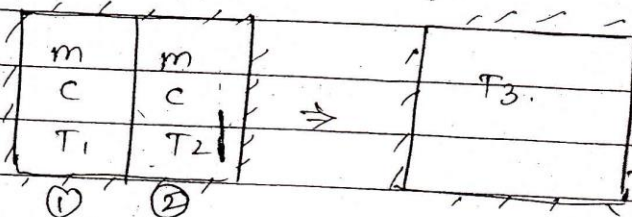
$$mc = 100/7$$

$$(ds)_{H_2O} = \frac{100}{7} \ln \left(\frac{280}{287} \right) = -0.3527$$

$$(ds)_{ref} = \frac{100}{276} = 0.3527$$

$$ds = 9.61 \times 10^{-3} \text{ kW/K} \quad \text{Ans}$$

Q.8 →



Let $T_1 > T_2$

Heat loss = Heat gain

$$mc(T_1 - T_3) = mc(T_3 - T_2)$$

$$T_1 - T_3 = T_3 - T_2$$

$$T_3 = \frac{T_1 + T_2}{2}$$

$$\Delta S_{\text{total}} = \Delta S_1 + \Delta S_2$$

$$= mc \ln\left(\frac{T_3}{T_1}\right) + mc \ln\left(\frac{T_3}{T_2}\right)$$

$$= mc \left[\ln\left(\frac{T_3}{T_1}\right) + \ln\left(\frac{T_3}{T_2}\right) \right]$$

$$= mc \ln \frac{T_3^2}{T_1 T_2}$$

$$= 2mc \ln \frac{T_3}{\sqrt{T_1 T_2}}$$

$$= 2mc \ln \left[\frac{T_1 + T_2}{2 \sqrt{T_1 T_2}} \right] \Delta$$

⇓

Q.2 we got

$$\Delta S_{\text{system}} = 2mc \ln \left(\frac{T_1 + T_2}{2 \sqrt{T_1 T_2}} \right)$$

$$= 2mc \ln \left[\frac{A \cdot m}{G \cdot m} \right]$$

We know $A \cdot m > G \cdot m$

$$\Rightarrow \ln \left(\frac{A \cdot m}{G \cdot m} \right) > 0$$

Hence $\Delta S_{\text{system}} > 0$

and mixing is adiabatic $\Rightarrow dQ = 0$

$$\Delta S_{\text{universe}} > 0$$

$$\Delta S_{\text{universe}} = \Delta S_{\text{sys}} + \Delta S_{\text{surroundings}}$$

$$> 0 \quad (dQ = 0)$$

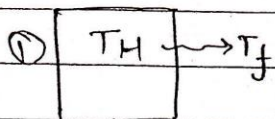
$$\Delta S_{\text{universe}} > 0$$

irreversible $\Rightarrow$ Mixing is an irreversible process.

Question (9) →

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$$w = Q_1 - Q_2$$

261) (SB)

$$w = mc [T_H - T_f - T_f + T_L]$$

$$Q_1 = mc (T_H - T_f)$$

$$w = mc [T_H + T_L - 2T_f]$$

①



we know

$$(\Delta S)_{\text{universe}} \geq 0$$

but engine is reversible

$$Q_2 = mc (T_f - T_L)$$

$$\Rightarrow (\Delta S)_{\text{universe}} = 0$$

$$(\Delta S)_{\text{system}} + (\Delta S)_{\text{surroundings}} = 0$$

Entropy is a property of system. system is going under cycle $\Rightarrow \Delta S_{\text{sys}} = 0$

$$\Rightarrow \Delta S_{\text{surroundings}} = 0$$

$$\Delta S_{\text{surroundings}} = \Delta S_1 + \Delta S_2$$

$$= mc \ln \left(\frac{T_f}{T_H} \right) + mc \ln \left(\frac{T_f}{T_L} \right) = 0$$

$$\Rightarrow mc \ln \frac{T_f^2}{T_H \cdot T_L} = 0$$

$$\ln \frac{T_f^2}{T_H \cdot T_L} = 0 \Rightarrow T_f^2 = T_H \cdot T_L$$

$$\Rightarrow T_f = \sqrt{T_H T_L} \quad \text{put in ①}$$

$$w = mc [T_H + T_L - 2\sqrt{T_H T_L}]$$

Q.12 $ds = \frac{Q}{T}$

$$\left[\frac{373}{T} \right]$$

$$\Rightarrow ds = \frac{-373}{373} = -1$$

HEAT PUMP

$$\downarrow 373J$$

(E)

Q.13 → same as Q.8

Q.14 → $Tds = du + pdv$, for solid $dv = 0$

$$Tds = du = mcdT \quad (m=1)$$

$$Tds = cdT$$

$$ds = \frac{c}{T} dT \Rightarrow \int_{S_1}^{S_2} ds = \int_{T_1}^{T_2} \frac{c}{T} dT$$

Eduwrite

15) for perfect gas
 $Tds = dH - vdp$
 $Tds = dH$
 $\frac{dH}{ds} = T$

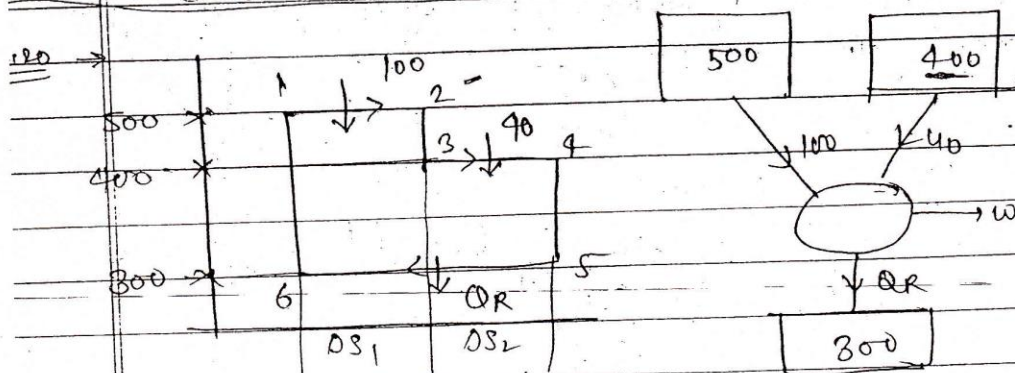
we know
 $\frac{dT}{ds} = \frac{T}{C_p}$
 $C_p = \frac{\gamma R}{\gamma - 1}$
 $\frac{dT}{ds} = \frac{(\gamma - 1)T}{\gamma R}$ ✓

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18) $ds = m c \ln \left(\frac{T_f}{T_i} \right)$
 $= 1 \times 1 \times \ln \left(\frac{873}{303} \right)$
 $ds = \ln \left(\frac{873}{303} \right)$ ✓ $ds = \ln \left(\frac{303}{873} \right) = -1.058$

19) $(\Delta S)_{sys} = C_v \ln \left(\frac{T_2}{T_1} \right) + R \ln \left(\frac{V_2}{V_1} \right)$
 free expansion of ideal gas $\rightarrow T_1 = T_2$
 $(\Delta S)_{sys} = C_v \ln(1) + R \ln \left(\frac{2V}{V} \right)$
 $= 0 + R \ln(2)$

$(\Delta S)_{surround} = 0$



$W = Q_1 - Q_R$

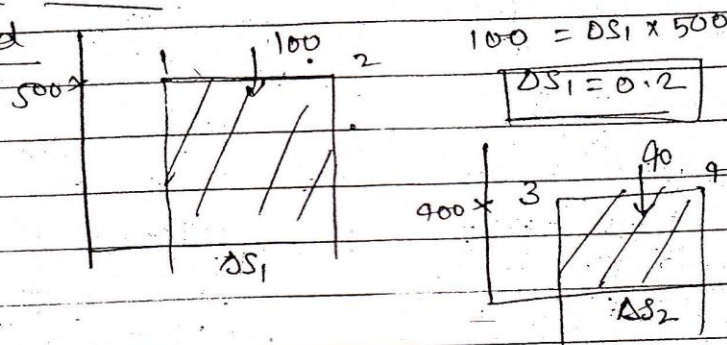
$W = 140 - Q_R$

$\oint_{rev} \frac{dQ}{T} = 0 \Rightarrow \frac{100}{500} + \frac{40}{400} - \frac{Q_R}{300} = 0$

$Q_R = 90$

$W = 140 - 90 = 50$

2nd method



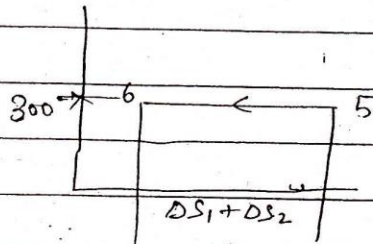
$$40 = \Delta S_2 \times 400$$

$$\Delta S_2 = 0.1$$

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26 (59)



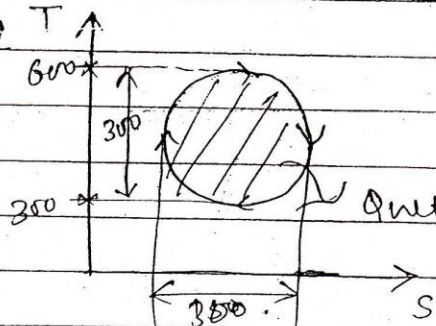
$$\Rightarrow 300 \times (\Delta S_1 + \Delta S_2) = Q_R$$

$$300 \times 0.3 = Q_R$$

$$Q_R = 90$$

$$\text{work done} = 100 + 40 - 90 = 50 \text{ kJ}$$

Q. 21



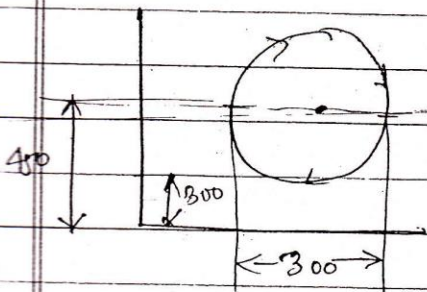
for a cycle

$$W_{\text{net}} = Q_{\text{net}}$$

$$Q_{\text{net}} = \text{Area}$$

$$= \frac{\pi}{4} (300)^2$$

$$W_{\text{net}} = \frac{\pi}{4} (300)^2$$



$$Q_{\text{net}} = (450 \times 300) + \frac{\pi}{4} (300)^2$$

$$= 135000 + \frac{\pi}{4} \times 90000$$

$$\eta = \frac{W_{\text{net}}}{Q_{\text{supplied}}}$$

$$\Rightarrow Q_{\text{net}} =$$

$$Q_{\text{sup}} = T_H \times \Delta S$$

$$W_{\text{net}} = \Delta S \times (T_H - T_C)$$

$$\eta = \frac{\Delta S (T_H - T_C)}{T_H \times \Delta S}$$

$$= \frac{T_H - T_C}{T_H}$$

$$= 0.5 (T_H - T_C)$$

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Eduwrite .

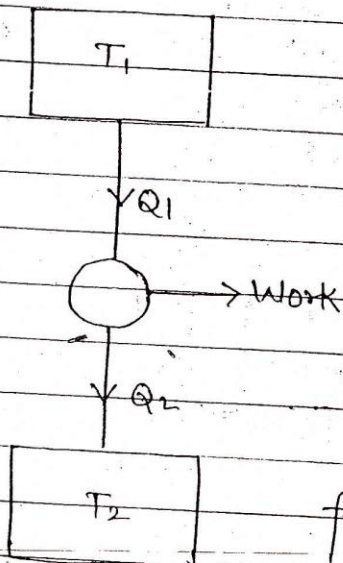
Signature

* AVAILABLE ENERGY, AVAILABILITY AND IRREVERSIBILITY *

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AVAILABLE ENERGY (AE)

The maximum possible work that can be obtained in a cycle is known as available energy. Let's consider a reversible heat engine -



Source temp. T_1 and heat received by engine Q_1 , both are fixed.

$$\eta_{\text{engine}} = \frac{W}{Q_1}$$

$$\eta_{\text{max}} = \frac{W_{\text{max}}}{Q_1}$$

for reversible engine

$$\eta_{\text{max}} = 1 - \frac{T_2}{T_1}$$

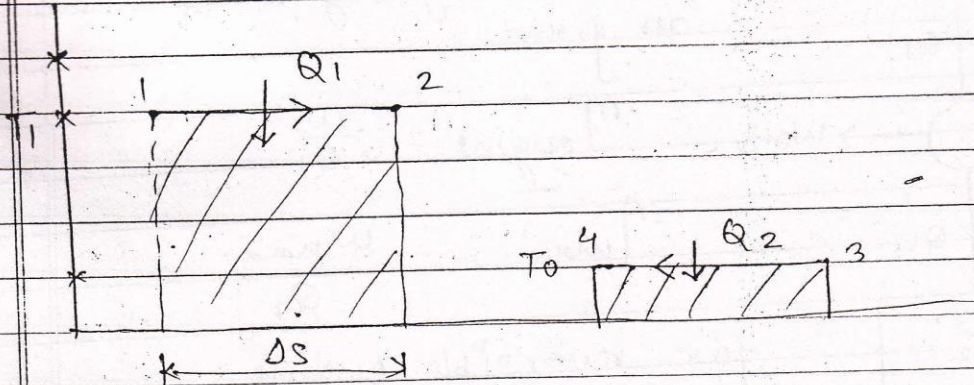
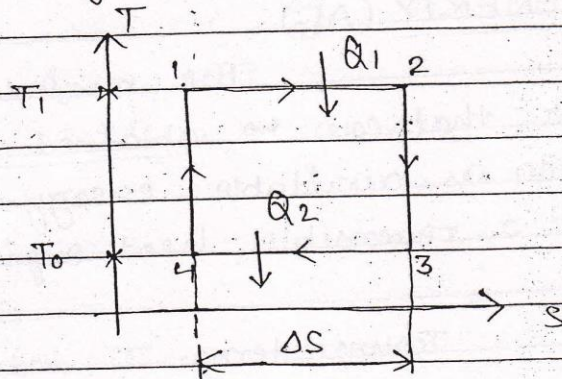
$$1 - \frac{T_2}{T_1} = \frac{W_{\text{max}}}{Q_1}$$

$$\Rightarrow W_{\text{max}} = Q_1 \left(1 - \frac{T_2}{T_1} \right)$$

This work can be further maximized if T_2 is minimum. The lowest possible temp. of heat rejection is that of surroundings (T_0).

$$\frac{W_{\text{max}}}{\text{possible}} \Rightarrow \boxed{AE = Q_1 \left(1 - \frac{T_0}{T_1} \right)}$$

* Now considering Carnot cycle on T-S diagram



$$Q_1 = T_1 \times \Delta S$$

$$Q_2 = T_0 \Delta S = \text{Heat Rejection}$$

$$\Delta S = \frac{Q_1}{T_1}$$

we know

$$AE = Q_1 \left(1 - \frac{T_0}{T_1} \right)$$

$$AE = Q_1 - T_0 \left(\frac{Q_1}{T_1} \right)$$

$$AE = Q_1 - T_0 \Delta S$$

OR $AE + T_0 \Delta S = Q_1$

⇒

$$AE + T_0 \Delta S = Q_1$$

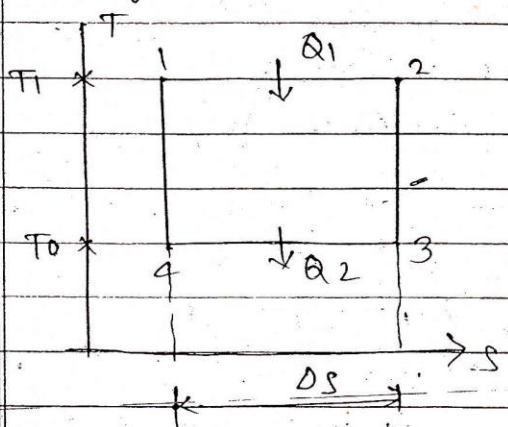
Max. work

min. heat
Rejection

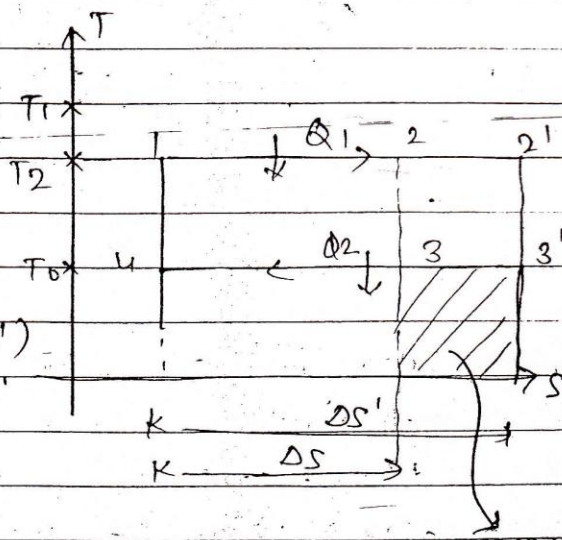
The minimum heat rejection is known as ~~244~~ (61) 'unavailable energy' (UAE), i.e. the area below T_0 , represents 'unavailable energy'.

Imp

* LOSS OF AVAILABLE ENERGY OR INCREASE IN UNAVAILABLE ENERGY WHEN HEAT IS TRANSFERRED THROUGH A FINITE TEMPERATURE DIFFERENCE
Considering Carnot cycle working in a temp. range T_1 & T_0



If Q_1 has to be kept constant and we want to change the source temp. to T_2 ($T_2 < T_1$)



We see, that Q_1 is same i.e. $(T_1 \times \Delta S) = (T_2 \times \Delta S')$

but there is an increase in UAE i.e.

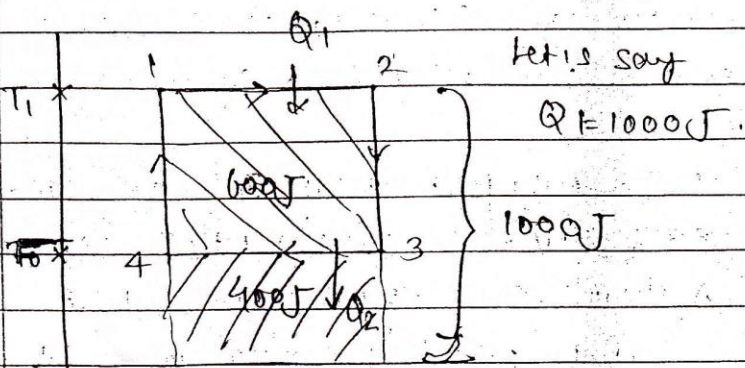
$$\text{Increase in UAE} = T_0 (\Delta S' - \Delta S) \quad \text{increase in UAE}$$

$$\text{Increase in UAE} = T_0 \left(\frac{Q_1}{T_2} - \frac{Q_1}{T_1} \right)$$

$$= T_0 Q_1 \left(\frac{T_1 - T_2}{T_1 T_2} \right)$$

$$\text{increase in UAE} = T_0 Q_1 \left(\frac{T_1 - T_2}{T_1 T_2} \right)$$

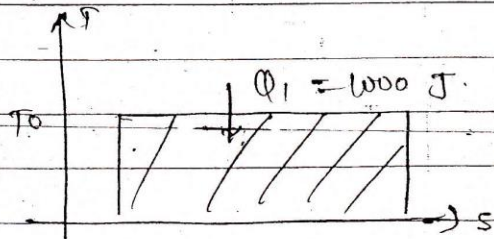
* Suppose we have thermal energy available at T_1 .



from fig. we see that out of $Q_1 = 1000 \text{ J}$

400 J is unavailable energy
and 600 J is available energy

If same Q_1 is available at T_0 .



All this energy is
unavailable energy as
it is corresponding to T_0

This can not be converted
into useful work.

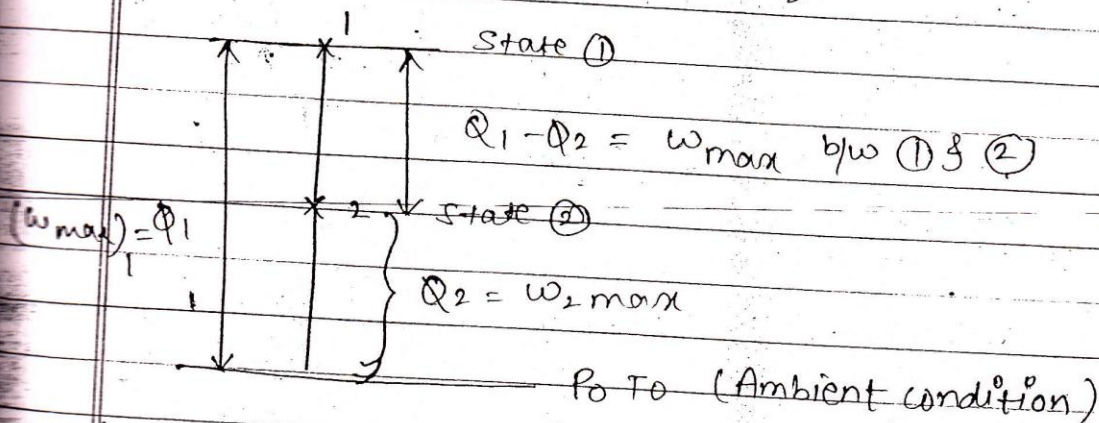
* According to I Law of T.D, thermal energy at higher temp and an equal amount of thermal energy at lower temperature have same ~~meaning~~ meaning, that means first law of thermodynamics visualises energy in quantity form. And therefore I law of thermodynamics is also known as Quantitative law.

* According to second law of T.D, thermal

Energy at higher temp. has greater significance when equal amount of thermal energy at lower temp. Because thermal energy at higher temp. is capable of producing more power. And hence 2nd law of T.D is also known as "Qualitative law".

* [AVAILABILITY] * The maximum work that can be obtained in a process in which the system comes into equilibrium with surroundings.

Maximum work b/w any two points is equal to = change in availability ($\Phi_1 - \Phi_2$)



* [MAXIMUM WORK IN A CLOSED SYSTEM] *

we know

$$dS_{\text{universe}} \geq 0$$

$$dS_{\text{universe}} = 0 \quad (\text{Reversible}).$$

$$dS_{\text{sys}} + dS_{\text{surr.}} = 0$$

$$\{ dS_{\text{sys}} = -dS_{\text{surr.}} \} \quad \text{--- (1)}$$

$$\Rightarrow \{ dQ_{sys} = -dQ_{surr} \} \quad \text{--- (2)}$$

Now $ds = \frac{dQ}{T}$

for surrounding (Temp = T_0)

$$dS_{surr} = \frac{dQ_{surr}}{T_0} \quad \text{--- (2)}$$

$$\{ dQ_{surr} = T_0 dS_{surr} \} \quad \text{--- (3)}$$

now, for a closed system

$$dQ_{sys} = dU_{sys} + dW_{sys}$$

$$dW_{sys} = dQ_{sys} - dU_{sys}$$

$$dW_{sys} = -dQ_{surr} - dU_{sys} \quad \text{[from (2)]}$$

$$dW_{sys} = -dQ_{surr} - dU_{sys}$$

$$\text{or } dW_{sys} = -T_0 dS_{surr} - dU_{sys} \quad \text{[from (3)]}$$

$$dW_{sys} = T_0 dS_{sys} - dU_{sys} \quad \text{[from (1)]}$$

integrating b/w (1) & (2)

$$W_{max} = T_0 (S_2 - S_1) - (U_2 - U_1)$$

**

$$W_{max} = (U_1 - U_2) - T_0 (S_1 - S_2)$$

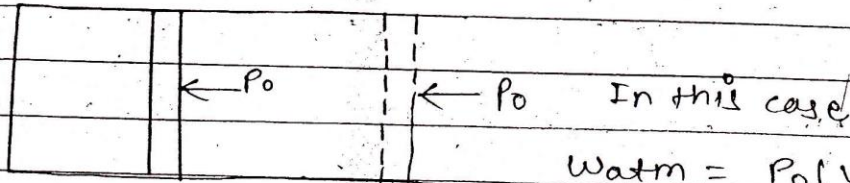
Maximum work for a closed system b/w any two points.

* we know

$$W_{max} = (U_1 - U_2) - T_0(S_1 - S_2)$$

→ max work for a closed system

Consider a closed system.



$$W_{atm} = P_0(V_2 - V_1)$$

$$\& W_{max} = (U_1 - U_2) - T_0(S_1 - S_2)$$

⇒ Max useful work = $W_{max} - W_{atm}$

$$\begin{aligned} W_{max, useful} &= (U_1 - U_2) - T_0(S_1 - S_2) - P_0(V_2 - V_1) \\ &= \underbrace{(U_1 - T_0 S_1 + P_0 V_1)}_{\Phi_1} - \underbrace{(U_2 - T_0 S_2 + P_0 V_2)}_{\Phi_2} \end{aligned}$$

$$W_{max, useful} = \Phi_1 - \Phi_2$$

↓ maximum useful work for a closed system.

where

$$\Phi_1 = U_1 - T_0 S_1 + P_0 V_1$$

$$\& \Phi_2 = U_2 - T_0 S_2 + P_0 V_2$$

Generally

$$\Phi = U - T_0 S + P_0 V$$

↓ Availability function for a closed system.

* [Maximum work for open system] *

for closed system, we started from

$$\{dQ = dU + dW\}$$

and

$$\text{got } W_{\max} = (U_1 - U_2) - T_0(S_1 - S_2)$$

Now for open system, we have equation

$$h_1 + \frac{C_p}{2} + g z_1 + q = h_2 + \frac{C_p}{2} + g z_2 + w_{c.v.}$$

assuming K.E & P.E changes are neglected

$$h_1 + q = h_2 + w_{c.v.}$$

$$\text{OR } q = (h_2 - h_1) + w$$

$$\{q = dh + dw\}$$

↳ for ~~closed~~ open system, this is only different from closed system's one
with $dU \rightarrow dh$

hence in the very same manner

$$(W_{\max})_{\text{open}} = (h_1 - h_2) - T_0(S_1 - S_2)$$

$$W_{\max} = (h_1 - T_0 S_1) - (h_2 - T_0 S_2)$$

$$W_{\max} = \phi_1 - \phi_2$$

$$\phi = h - T_0 S \rightarrow \text{Availability function for open system}$$

$$\Rightarrow W_{\max, \text{ useful}} = W_{\max} = \phi_1 - \phi_2$$

Edurite

↳ Because no atm work component here

IRREVERSIBILITY (I)

26/11/20 (24)

Irreversibility (I) is defined as the difference of maximum work & actual work.

$$I = w_{\max} - w_{\text{actual}}$$

$$I = T_0 [(\Delta S)_{\text{sys}} + (\Delta S)_{\text{surr.}}]$$

$$I = T_0 (\Delta S)_{\text{universe}}$$

as $T_0 > 0$ & $(\Delta S)_{\text{universe}} \geq 0$

$$\Rightarrow I \geq 0 \text{ always.}$$

Gouy-Stodola theorem

$$\text{as } I = T_0 (\Delta S)_{\text{universe}}$$

↳ constant

$$I \propto (\Delta S)_{\text{univ.}}$$

In rate form $\dot{I} \propto (\dot{\Delta S})_{\text{univ.}}$

* GIBB'S FUNCTION (G)

$$G = H - TS$$

* HELMHOLTZ FUNCTION (F)

$$F = U - TS$$

⇒ Diff. b/w G_1 & G_2 gives → open sys. max work
 ⇒ Diff. b/w F_1 & F_2 gives → closed sys max. work

Eduwrite

Question ① → Find the loss of A.E. associated in the transfer of 1000 KJ of heat from a constant temp. system at 600 K to another at 400 K and ambient temp. is 300 K.

Solⁿ

loss of A.E. = Increase in U.A.E

$$T_1 = 600 \text{ K}$$

$$T_2 = 400 \text{ K}$$

$$Q_1 = 1000 \text{ KJ}$$

$$T_0 = 300 \text{ K}$$

$$\text{increase in U.A.E} = T_0 Q_1 \left(\frac{T_1 - T_2}{T_1 T_2} \right)$$

$$= 300 \times 1000 \left(\frac{600 - 400}{600 \times 400} \right)$$

$$= \frac{30}{24} \times 200$$

$$= \frac{30 \times 200}{24}$$

$$= 250 \text{ KJ. Ans}$$

Q → ② For steady flow process from state 1 to state 2 enthalpy changes from $h_1 = 400 \text{ KJ/kg}$ to $h_2 = 100 \text{ KJ/kg}$. And entropy changes from $S_1 = 1.1 \text{ KJ/kg-K}$ to $S_2 = 0.7 \text{ KJ/kg-K}$. And the ambient temp (T_0) is 300 K. Find the change in availability

Solⁿ

$$\rightarrow w_{\max} = (h_1 - h_2) - T_0(S_1 - S_2)$$

$$= (400 - 100) - 300(1.1 - 0.7)$$

Change in
availability

$$= 300 [1 - 0.4]$$

$$= 300 \times 0.6$$

$$= 180 \text{ KJ. Ans}$$

Fig

Question (3) → Steam flows steadily in an adiabatic turbine. The enthalpy at the entrance is ~~400~~ 4142 KJ/Kg and the enthalpy at exit is 2505 KJ/Kg. The values of flow availability at the inlet and exit are ~~1707~~ 1787 KJ/Kg and 140 KJ/Kg. The ambient temp is 300K. Find

- Actual work
- Max work
- change in entropy of system

Solution →

$$h_1 = 4142 \text{ KJ/Kg}, \quad h_2 = 2505 \text{ KJ/Kg}$$

$$\phi_1 = 1787 \text{ KJ/Kg}, \quad \phi_2 = 140 \text{ KJ/Kg}$$

$$(i) \quad w_{\max} = \phi_1 - \phi_2$$

$$= 1787 - 140$$

$$= 1647 \text{ KJ/Kg} \quad \underline{\text{Ans}}$$

(ii) for actual work

for open system, SFEE

$$h_1 + \frac{C_1^2}{2} + g z_1 + q_1 = h_2 + \frac{C_2^2}{2} + g z_2 + w$$

$$h_1 = h_2 + w$$

$$w = h_1 - h_2 \quad (\text{actual work})$$

$$w_{\text{actual}} = 4142 - 2585 = 1557 \text{ KJ/Kg} \quad \underline{\text{Ans}}$$

(ii) we know $\left[w_{\max} \right]_{\text{open}} = (h_1 - h_2) - T_0 (s_1 - s_2)$

$$w_{\max} = 1557 - T_0 (s_1 - s_2)$$

$$T_0 (s_2 - s_1) = 1647 - 1557$$

Eduwrite

$$s_2 - s_1 = 0.3 \text{ KJ/Kg-K} \quad \underline{\text{Ans}}$$

[IES-2008]

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Q (21) → ~~Steam~~ ~~flows~~ ~~steadily~~ ~~in an~~ ~~adi~~

Find the irreversibility associated with the expansion of air through a very small opening from pressure & temp of 8 bar & 600 K to a pressure of 1.2 bar.

Assume air to be an ideal gas & $T_0 = 298 \text{ K}$.

Solution → ^{air is} As the ^{air is} flowing through a very small opening the process undergone by the fluid is throttling process.

We know

$$I = T_0 (ds)_{\text{universe}}$$

$$I = T_0 [(ds)_{\text{sys}} + (ds)_{\text{surr}}]$$

Conditions associated with throttling process

are (i) No heat transfer $\Rightarrow (ds)_{\text{surr}} = 0$

(ii) $h_1 = h_2$

Now $I = T_0 (ds)_{\text{sys}}$

$$\begin{aligned} h_1 &= h_2 \\ c_p T_1 &= c_p T_2 \end{aligned} \quad \left. \begin{array}{l} \text{for air (ideal gas)} \\ h = m c_p T \end{array} \right\}$$

$$T_1 = T_2$$

Entropy change for air = $(ds)_{\text{sys}}$

$$(ds)_{\text{sys}} = c_p \ln \frac{T_2}{T_1} - R \ln \left(\frac{P_2}{P_1} \right)$$

$$= c_p \ln (1) - R \ln \left(\frac{P_2}{P_1} \right)$$

$$= 0 + R \ln \left(\frac{P_1}{P_2} \right)$$

$$I = T_0 (ds)_{\text{system}}$$

259 (68)

$$I = R T_0 \ln \left(\frac{P_1}{P_2} \right)$$

$$\{ I = 162.25 \text{ KJ / kg} \} \text{ Ans.}$$

Question (5) → An adiabatic cylinder of 10 m^3 volume is divided into two compartments A & B. Each of volume 6 m^3 and 4 m^3 by a thin sliding partition. Initially the compartment A is filled with air at 6 bar, 600 K, while there is vacuum in compartment B. Suddenly the partition is removed and the fluid in compartment A expands freely and fills both the compartments. Calculate the irreversibility of the process in KJ. Take $P_0 = 1 \text{ bar}$ & $T_0 = 300 \text{ K}$.

Soln →

6 bar			
A 600 K	B	⇒	$V_2 = 10 \text{ m}^3$
6 m^3	4 m^3		

free expansion is taking place

$$I = T_0 (ds)_{\text{univ}}$$

$$I = T_0 (ds)_{\text{sys}} \quad [\because (ds)_{\text{surr}} = 0]$$

as $Q = 0$

work done = 0

we know, when free expansion takes place

$$T_i = T_f \rightarrow \text{for ideal gas.}$$

$$(ds)_{\text{system}} = \frac{P_1 V_1}{P_2 V_2} = \frac{P_1 V_1}{P_2 V_2}$$

$$6 \times 6 = P_2 \times 10$$

$$P_2 = 3.6 \text{ bar}$$

$$(ds)_{\text{ideal gas}} = C_v \ln \left(\frac{T_2}{T_1} \right) + R \ln \left(\frac{V_2}{V_1} \right)$$

$$= 0 + 0.287 \times \ln \left(\frac{10}{6} \right)$$

$$(ds)_{\text{system}} = 0.1466 \text{ kJ/kg}$$

$$pV = mRT$$

$$m = \frac{(600 \text{ kPa}) \times 6 \text{ m}^3}{0.287 \frac{\text{kJ}}{\text{kg-K}} \times 600 \text{ K}}$$

$$m = 20.9 \text{ kg}$$

$$(ds) = m(ds)$$

$$ds = 20.9 \times 0.1466 = 3.064 \text{ kJ}$$

$$I = T_0(ds)_{\text{system}}$$

$$I = 919.2 \text{ kJ} \quad \underline{\text{Ans}}$$

Question (6) A solid aluminium sphere of 1 m diameter and initially at 200°C is cooled to an atm. temp. of 25°C . The density of aluminium is 2700 kg/m^3 and specific heat is 0.9 kJ/kg-K . Find the irreversibility associated with this process!

Soln

$$T_1 = 200 + 273 = 473 \text{ K}$$

$$T_2 = 25^\circ\text{C} = 298 \text{ K}$$

$$c = 0.9 \text{ kJ/kg-K}$$

$$I = T_0(ds)_{\text{univ.}}$$

$$(ds)_{\text{univ.}} = (ds)_{\text{sys}} + (ds)_{\text{surr.}}$$

$$(ds)_{sys} = mc \ln \left(\frac{T_f}{T_i} \right)$$

22/ (67)

$$m = \text{Volume} \times \text{density}$$

$$= 2700 \times 4\pi \text{ m}^3$$

$$= 2700 \times \frac{4 \times \pi}{3} (1.25 \times 10^{-4})$$

$$m = \frac{4.2411 \text{ kg}}{3} = 1.4137$$

$$(ds)_{sys} = 1.4137 \times 0.9 \ln \left(\frac{298}{473} \right)$$

$$(ds)_{sys} = -1.4137 \times 0.9 \ln \left(\frac{473}{298} \right) = -0.5878$$

$$(ds)_{surr} = (ds)_{sink} = \frac{Q}{T_{surr}}$$

$$Q = m \cdot c \cdot \Delta T$$

$$= 4.2411 \times 0.9 \times (473 - 298) \times 1.4137$$

$$Q = 667.9 \text{ J} \quad 222.657 \text{ KJ}$$

$$Q = 667.9 \text{ J}$$

$$(ds)_{surr} = \frac{667.9}{298} = 2.24$$

$$Q = 222.65 \text{ KJ}$$

$$(ds)_{surr} = \frac{222.65}{298} = 0.7469$$

$$I = T_0 (ds)_{univ}$$

$$I = T_0 [-0.5878 + 0.7469]$$

$$\{I = 47.47 \text{ KJ}\} \text{ Am}$$

Q. (7) Find the max. work per kg. of air that can be obtained in a piston cylinder arrangement. Air expands from initial state of 9 bar, 400 K to the final state of 1.5 bar, 300 K. $P_0 = 1 \text{ bar}$ and $T_0 = 288 \text{ K}$. Also find the max. useful work.

Solⁿ → $P_1 = 9 \text{ bar}$, $T_1 = 400 \text{ K}$
 $P_2 = 1.5 \text{ bar}$, $T_2 = 300 \text{ K}$

~~$P_1 V_1^\gamma = P_2 V_2^\gamma$~~

$[W_{\max}]_{\text{closed sys}} = U_1 - U_2 - T_0 (S_1 - S_2)$

for ideal gas

$W_{\max} = C_v (T_1 - T_2) - T_0 (S_1 - S_2)$

$S_2 - S_1 = C_p \ln \left(\frac{T_2}{T_1} \right) + R \ln \left(\frac{P_2}{P_1} \right)$

$S_2 - S_1 = 1.005 \ln \left(\frac{300}{400} \right) - 0.287 \ln \left(\frac{1.5}{9} \right)$

$S_2 - S_1 = 0.225 \text{ KJ/kg}$

$W_{\max} = C_v (T_1 - T_2) - T_0 (S_1 - S_2)$

$= 0.718 (400 - 300) - 288 (-0.225)$

$= 0.718 \times 100 + 288 \times 0.225$

$W_{\max} = 136.63 \text{ KJ/kg}$ Ans

we know, $W_{\max, \text{useful}} = W_{\max} - \underbrace{P_0 (V_2 - V_1)}_{W_{\text{atm}}}$

$$PV = mRT$$

taking $m = 1 \text{ kg}$.

$$V_1 = \frac{RT_1}{P_1} = \frac{0.287 \times 400}{900}$$

$$\{V_1 = 0.1275 \text{ m}^3\}$$

$$V_2 = \frac{RT_2}{P_2} = \frac{0.287 \times 300}{1.50}$$

$$\{V_2 = 0.574 \text{ m}^3\}$$

$$W_{\text{max, useful}} = 136.63 - 100 \times (0.574 - 0.1275) \\ = 91.98 \text{ kJ/kg.}$$

$$\boxed{W_{\text{max, useful}} = 91.98 \text{ kJ/kg}} \quad \text{Ans}$$

CHAPTER- 7 ^{***} IES

* THERMODYNAMIC RELATIONS *

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THEOREM - (1)

The equation
 $dz = Mdx + Ndy$ is exact
 if $\left[\left(\frac{\partial M}{\partial y} \right)_x = \left(\frac{\partial N}{\partial x} \right)_y \right]$

THEOREM (2) if $f = \phi(x, y, z)$

then $\left[\left(\frac{\partial x}{\partial y} \right)_f \left(\frac{\partial y}{\partial z} \right)_f \left(\frac{\partial z}{\partial x} \right)_f = 1 \right]$

THEOREM (3) if $z = \phi(x, y)$

then $\left[\left(\frac{\partial x}{\partial y} \right)_z \left(\frac{\partial y}{\partial z} \right)_x \left(\frac{\partial z}{\partial x} \right)_y = -1 \right]$

**

[Maxwell's equation]

* we know, from the combination of 1st, 2nd & 3rd law of T.D.

$$Tds = du + pdv$$

$$du = Tds - pdv$$

Comparing with $dz = Mdx + Ndy$.

u is a property of system $\Rightarrow$ Exact differential

$$\left(\frac{\partial M}{\partial y} \right)_x = \left(\frac{\partial N}{\partial x} \right)_y$$

$$\begin{matrix} M = T, & x = S \\ N = -P, & y = V \end{matrix}$$

$$\left[\left(\frac{\partial T}{\partial V} \right)_S = \left(\frac{\partial P}{\partial S} \right)_V \right]$$

— (1)

I Maxwell's eqⁿ.

* we know $Tds = dH - vdp$

$$dH = Tds + vdp$$

$$dz = Mdx + Ndy$$

Comparing $M = T$, $x = s$, $N = v$, $y = p$.

$H \rightarrow$ Property of system $\rightarrow$ Exact diff.

$$\left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y$$

$$\boxed{\left(\frac{\partial T}{\partial p}\right)_s = \left(\frac{\partial v}{\partial s}\right)_p} \quad \text{--- (2) 2nd Maxwell's eqn}$$

* we know, Gibbs function is given as

$$G = H - TS$$

Differentiating

$$dG = dH - d(TS)$$

$$dG = dH - Tds - sdT$$

$$dG = (dH - Tds) - sdT$$

$$\left\{ \begin{array}{l} \text{we know } Tds = dH - vdp \\ dH - Tds = vdp \end{array} \right.$$

$\rightarrow$ hence

$$dG = vdp - sdT$$

Comparing with $dz = Mdx + Ndy$

$$M = v, x = p, N = -s, y = T$$

$$\left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y$$

$$\boxed{\left(\frac{\partial v}{\partial T}\right)_p = \left(\frac{-\partial s}{\partial p}\right)_T}$$

* We know Helmholtz function

$$F = U - TS$$

$$dF = dU - Tds - sdT$$

$$dF = (dU - Tds) - sdT$$

We know $Tds = dU + PdV$

$$dU - Tds = -PdV$$

So $dF = -PdV - sdT$

Comparing with $dz = Mdx + Ndy$

$$M = -P, \quad x = V, \quad N = -S, \quad y = T$$

$$\left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y$$

$$\left(-\frac{\partial P}{\partial T}\right)_V = \left(-\frac{\partial S}{\partial V}\right)_T$$

OR $\boxed{\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T}$

or Maxwell's eqns are,

$$\boxed{(1) \left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V} \quad \text{--- (1)}$$

$$\boxed{(2) \left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P} \quad \text{--- (2)}$$

$$\boxed{(3) \left(\frac{\partial V}{\partial T}\right)_P = -\left(\frac{\partial S}{\partial P}\right)_T} \quad \text{--- (3)}$$

$$\boxed{(4) \left(\frac{\partial P}{\partial T}\right)_T = \left(\frac{\partial S}{\partial V}\right)_T} \quad \text{--- (4)}$$

Equating both the equation

$$C_v dT + T \left(\frac{\partial P}{\partial T} \right)_V dv = C_p dT - T \left(\frac{\partial v}{\partial T} \right)_P dP$$

$$T \left(\frac{\partial P}{\partial T} \right)_V dv + T \left(\frac{\partial v}{\partial T} \right)_P dP = (C_p - C_v) dT$$

OR
$$dT = \frac{T}{C_p - C_v} \left(\frac{\partial P}{\partial T} \right)_V dv + \frac{T}{(C_p - C_v)} \left(\frac{\partial v}{\partial T} \right)_P dP$$

$$dT = \left(\frac{T}{C_p - C_v} \right) \left[\left(\frac{\partial P}{\partial T} \right)_V dv + \left(\frac{\partial v}{\partial T} \right)_P dP \right] \quad (1)$$

Hence $T = f(P, v)$

$$dT = \left(\frac{\partial T}{\partial P} \right)_V dP + \left(\frac{\partial T}{\partial v} \right)_P dv \quad (2)$$

Comparing (1) & (2)

$$\frac{T}{C_p - C_v} \left(\frac{\partial P}{\partial T} \right)_V = \left(\frac{\partial T}{\partial P} \right)_V$$

$$\frac{T}{C_p - C_v} \left(\frac{\partial v}{\partial T} \right)_P = \left(\frac{\partial T}{\partial P} \right)_V$$

$$C_p - C_v = T \left(\frac{\partial v}{\partial T} \right)_P \left(\frac{\partial P}{\partial T} \right)_V \quad (3)$$

*** (Derivation)

→ valid for any ^{IES} gas
 → But $C_p - C_v \neq R$ for a gas other than ideal gas.

from (3)

$$C_p - C_v = T \left(\frac{\partial v}{\partial T} \right)_P \left(\frac{\partial P}{\partial T} \right)_V. \quad \text{--- (3)}$$

So we can write

$$P = f(P, V).$$

from 3rd theorem.

$$\left(\frac{\partial P}{\partial V} \right)_T \left(\frac{\partial V}{\partial T} \right)_P \left(\frac{\partial T}{\partial P} \right)_V = -1.$$

$$\text{OR.} \quad \left(\frac{\partial P}{\partial T} \right)_V = - \left(\frac{\partial V}{\partial P} \right)_T \left(\frac{\partial T}{\partial V} \right)_P$$

$$\text{OR} \quad \left(\frac{\partial P}{\partial T} \right)_V = - \left(\frac{\partial P}{\partial V} \right)_T \left(\frac{\partial V}{\partial T} \right)_P.$$

Putting in (3).

$$C_p - C_v = T \left(\frac{\partial v}{\partial T} \right)_P \left[- \left(\frac{\partial P}{\partial V} \right)_T \left(\frac{\partial v}{\partial T} \right)_P \right]$$

$$= - T \left[\left(\frac{\partial v}{\partial T} \right)_P \right]^2 \left(\frac{\partial P}{\partial V} \right)_T$$

V.V. Imp (ES) 2012

$$C_p - C_v = - T \left[\left(\frac{\partial v}{\partial T} \right)_P \right]^2 \left(\frac{\partial P}{\partial V} \right)_T. \quad \text{--- (4)}$$

* $\left(\frac{\partial P}{\partial V} \right)_T$ is always negative and hence

$(C_p - C_v)$ is always +ve.

Therefore

$$C_p - C_v > 0$$

$$[C_p > C_v]$$

* Coefficient of volume expansivity (β)

$$\left\{ \beta = \frac{1}{V} \left(\frac{\partial V}{\partial T} \right)_P \right\}$$

* Isothermal compressibility (K_T)

$$K_T = -\frac{1}{V} \left(\frac{\partial V}{\partial P} \right)_T$$

we know

$$C_p - C_v = -T \left[\left(\frac{\partial V}{\partial T} \right)_P \right]^2 \left(\frac{\partial P}{\partial V} \right)_T$$

but

$$\left(\frac{\partial V}{\partial P} \right)_T = -K_T \cdot V \quad \left(\frac{\partial V}{\partial T} \right)_P = \beta V$$

$$\text{OR} \quad \left(\frac{\partial P}{\partial V} \right)_T = -\frac{1}{K_T \cdot V}$$

$$C_p - C_v = -T \left[(\beta V)^2 \right] \left[-\frac{1}{K_T \cdot V} \right]$$

$$C_p - C_v = \frac{T V \beta^2}{K_T}$$

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[ENERGY EQUATION]

we know

$$TdS = dU + PdV$$

$$dU = TdS - PdV$$

from 1st Tds equation

$$TdS = C_v dT + T \left(\frac{\partial P}{\partial T} \right)_V dV$$

$$\Rightarrow dU = C_v dT + T \left(\frac{\partial P}{\partial T} \right)_V dV - PdV$$

$$dU = C_v dT + \left[T \left(\frac{\partial P}{\partial T} \right)_V - P \right] dV \quad \text{--- (1)}$$

↳ Energy equation (valid for any gas).

* for ideal gas

$$PV = mRT$$

$$P = \left(\frac{mR}{V} \right) T \quad \text{--- (2)}$$

if volume is constant $\left(\frac{\partial P}{\partial T} \right)_V = \frac{mR}{V}$

$$\text{i.e. } \left(\frac{\partial P}{\partial T} \right)_V = \frac{mR}{V}$$

$$\text{but } PV = mRT \Rightarrow \frac{mR}{V} = \frac{P}{T}$$

$$\Rightarrow \left(\frac{\partial P}{\partial T} \right)_V = \frac{P}{T}$$

$$\text{OR } \left\{ T \left(\frac{\partial P}{\partial T} \right)_V - P \right\} = 0$$

Putting in (1)

$$dU = C_v dT$$

$$dU = C_v dT \quad \text{for ideal gas only}$$

* Energy equation

$$dU = C_v dT + \left[T \left(\frac{\partial P}{\partial T} \right)_V - P \right] dV \quad \text{--- (1)}$$

$$U = f(T, V)$$

$$dU = \left(\frac{\partial U}{\partial T} \right)_V dT + \left(\frac{\partial U}{\partial V} \right)_T dV \quad \text{--- (2)}$$

Comparing (1) & (2)

$$\left\{ C_v = \left(\frac{\partial U}{\partial T} \right)_V \right\}, \quad \left\{ T \left(\frac{\partial P}{\partial T} \right)_V - P = \left(\frac{\partial U}{\partial V} \right)_T \right\}$$

for ideal gas

$$T \left(\frac{\partial P}{\partial T} \right)_V - P = 0$$

$$\Rightarrow \left[\left(\frac{\partial U}{\partial V} \right)_T = 0 \right] \rightarrow \text{for ideal gas}$$

* for an ideal gas, the internal energy does not change with volume, i.e. internal energy of an ideal gas is independent of volume.

$$\text{Now } U = \phi(T, P, V)$$

$$\text{OR } T = g(U, P, V)$$

from 2nd theorem

$$\left(\frac{\partial U}{\partial P} \right)_T \left(\frac{\partial P}{\partial V} \right)_T \left(\frac{\partial V}{\partial U} \right)_T = 1$$

$$\text{OR } \left(\frac{\partial U}{\partial P} \right)_T \left(\frac{\partial P}{\partial V} \right)_T = \left(\frac{\partial U}{\partial V} \right)_T$$

$$\text{for an ideal gas } \left(\frac{\partial U}{\partial V} \right)_T = 0$$

$$\Rightarrow \left(\frac{\partial U}{\partial P} \right)_T \left(\frac{\partial P}{\partial U} \right)_T = 0$$

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↓
always -ve ie $\neq 0$

$$\Rightarrow \left[\left(\frac{\partial U}{\partial P} \right)_T = 0 \right] \rightarrow \text{Valid for ideal gas}$$

CHAPTER-6

* PROPERTIES OF PURE SUBSTANCES *

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* Suppose we have to convert water (20°C) to steam (200°C)

Stage I

H_2O at 20°C $\rightarrow$ H_2O at 100°C .

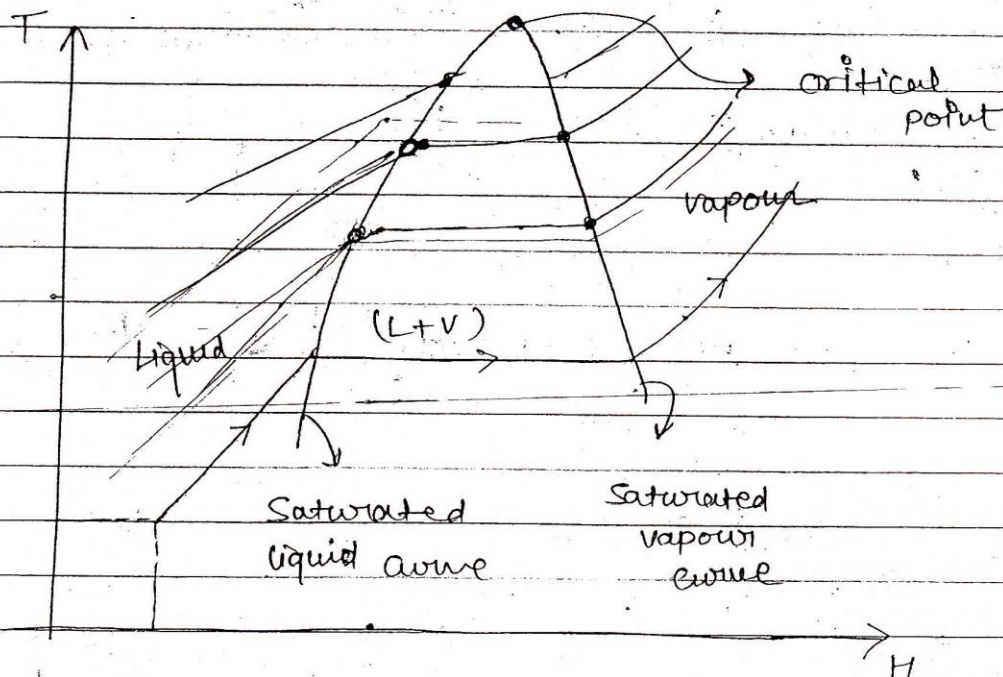
Stage II

H_2O at 100°C $\rightarrow$ steam at 100°C

Stage III

Steam at 100°C $\rightarrow$ steam at 200°C .

* [Pressure is constant throughout the process]

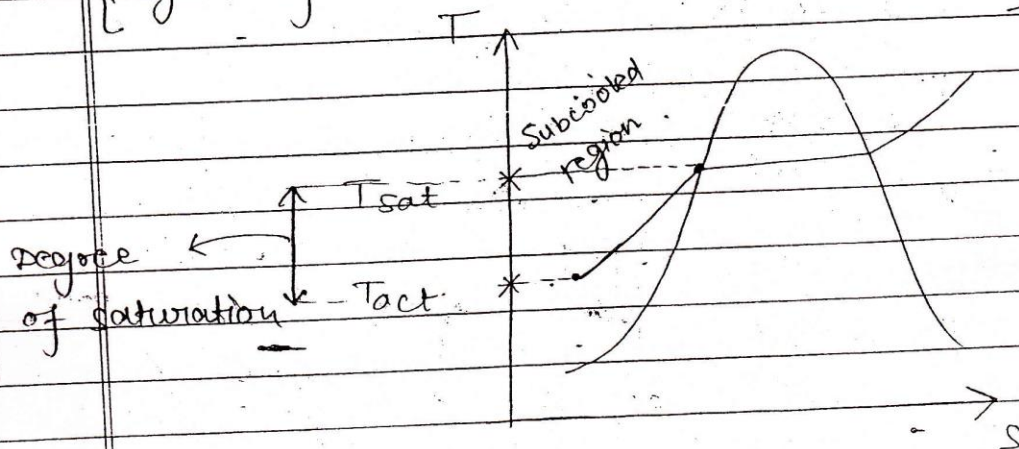


CRITICAL POINT $\rightarrow$ It is the point at which the saturated liquid and saturated vapour curve meets.

* SUBCOOLED / UNDERCOOLED REGION →

It is a region in which the actual temp is less than saturation temp corresponding to ~~sat~~ that pressure

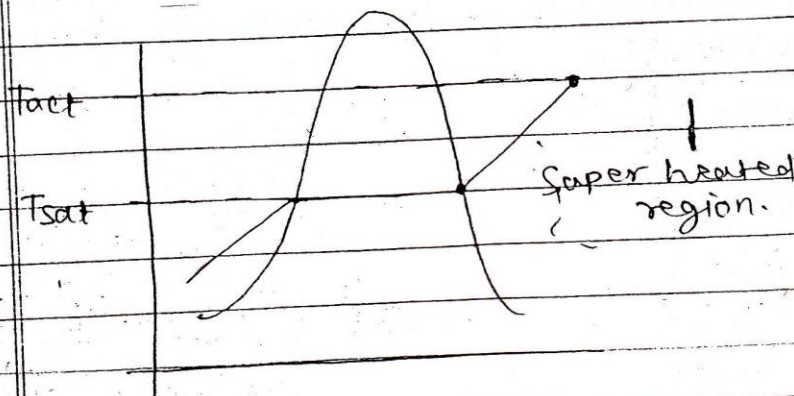
$$* \{ \text{Degree of saturation} = T_{\text{sat}} - T_{\text{actual}} \}$$



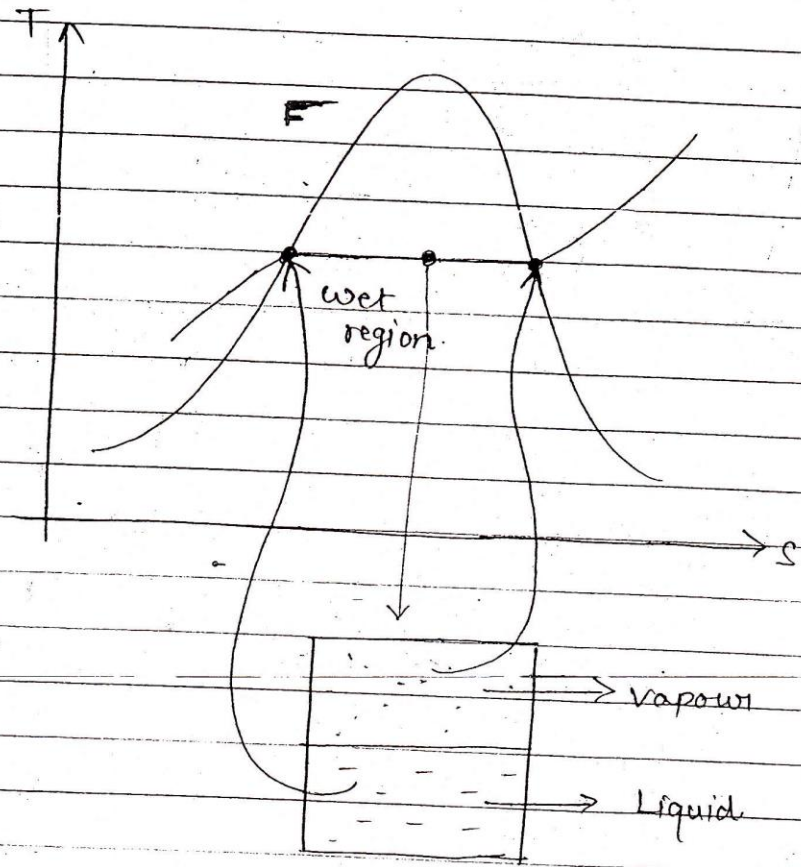
* SUPERHEATED REGION

It is a region in which the actual temperature is more than saturation temperature corresponding to that pressure

$$* \{ \text{Degree of superheat} = T_{\text{actual}} - T_{\text{saturation}} \}$$



* WET REGION It is a region in which liquid & vapour exist in equilibrium. And the corresponding temp is equal to saturation temperature.



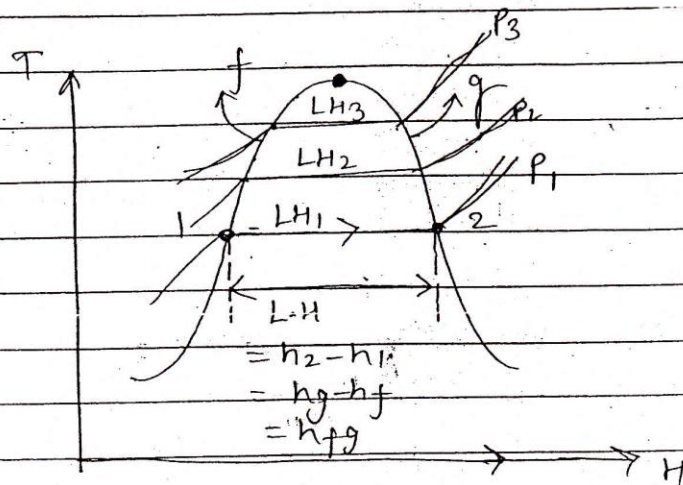
SENSIBLE HEAT → The heat transfer associated due to temperature change is known as sensible heat.

$$\{ \text{Sensible heat} = m \cdot c \cdot \Delta T \}$$

LATENT HEAT → The heat transfer associated during phase change is known as latent heat.

we know

Latent heat = phase change heat



L.H. = phase change heat

but phase change occurs at constant pressure

$$LH = Q_p = dh \quad \left\{ \begin{array}{l} \text{for closed system} \\ \text{Constant pr. heat supplied} \end{array} \right.$$

$$LH = h_2 - h_1$$

$$LH = h_g - h_f = h_{fg}$$

* from fig. we can see - as pressure increases, L.H. decreases.

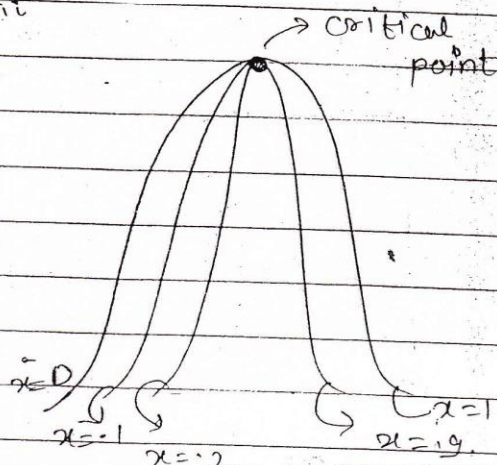
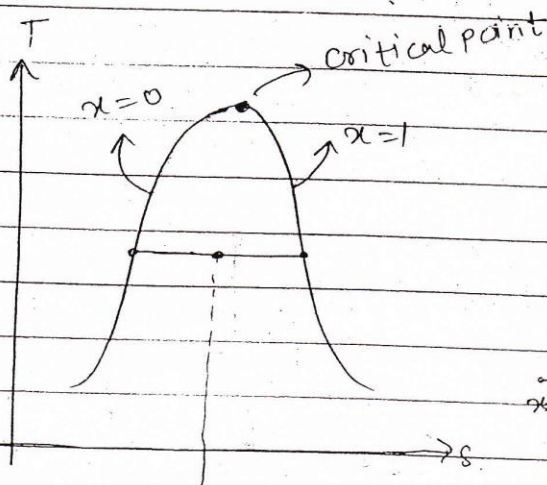
* At critical point, Latent heat of vapourisation is zero.

h_g : enthalpy of saturated vapour

h_f : " " " " liquid.

It is defined as the ratio of mass of vapour to the total mass of the mixture.

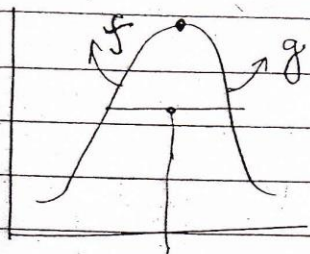
$$x = \frac{m_v}{m_v + m_e}$$



⇒ All dryness fraction lines converge at critical point. (It is not defined at critical point).

* SPECIFIC VOLUME OF MIXTURE *

we know volume of mixture $V = V_1 + V_2$



$$V = V_v + V_d$$

~~$$mv = m_v v_v + m_l v_l$$~~

$$u = \frac{m_v u_v + m_e u_e}{m}$$

Eduwritte

$$v = \frac{m_v}{m} v_v + \frac{m_l}{m} v_l$$

$$v = \left(\frac{m_v}{m_v + m_l} \right) v_v + \frac{(m - m_v)}{m} v_l$$

$$v = x v_v + \left(1 - \frac{m_v}{m_l + m_v} \right) v_l$$

$$v = x v_v + (1 - x) v_l$$

$$\Rightarrow v = x (v_v - v_l) + v_l$$

$$v = v_l + x (v_v - v_l)$$

$$\boxed{v = v_f + x (v_g - v_f)} \quad \text{--- (1)}$$

* Similarly we have

$$\text{as } H = H_g + H_f$$

$$\boxed{h = h_f + x (h_g - h_f)} \quad \text{--- (2)}$$

$$* S = S_g + S_f$$

$$\boxed{s = s_f + x (s_g - s_f)} \quad \text{--- (3)}$$

$$* U = U_g + U_f$$

$$\boxed{u = u_f + x (u_g - u_f)} \quad \text{--- (4)}$$

* All the four equations are valid in wet region.

[ENTHALPY AT VARIOUS POINTS]

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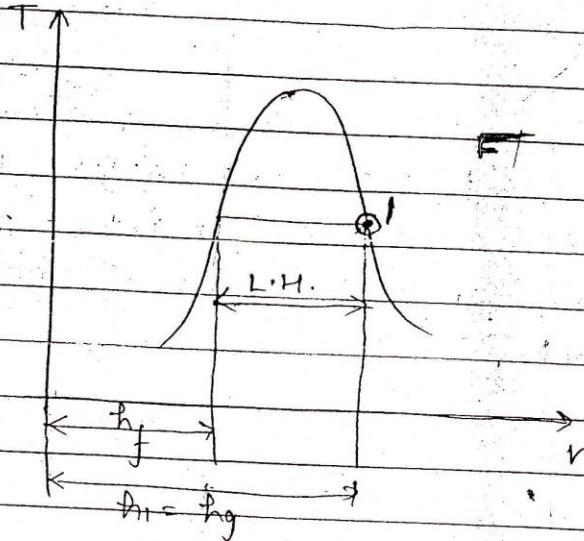
CASE - 1

when the point is on saturated vapour curve.

$$h_1 = h_g + L.H.$$

$$h_g = h_f + L.H.$$

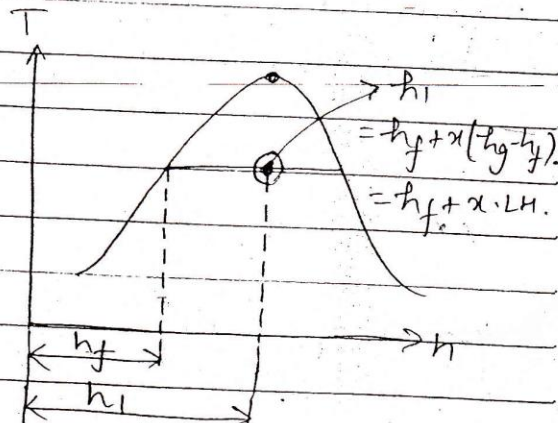
$$L.H. = h_g - h_f$$



CASE (2) [when the point is in wet region]

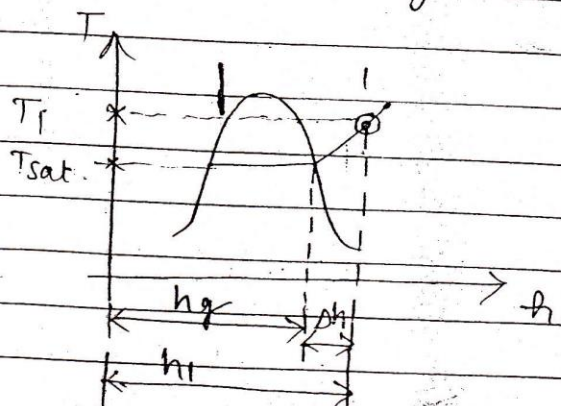
$$h_1 = h_f + x (h_g - h_f)$$

$$h_1 = h_f + x (L.H.)$$



CASE (3) when the point is in superheated region.

$$h_1 = h_g + \Delta h$$

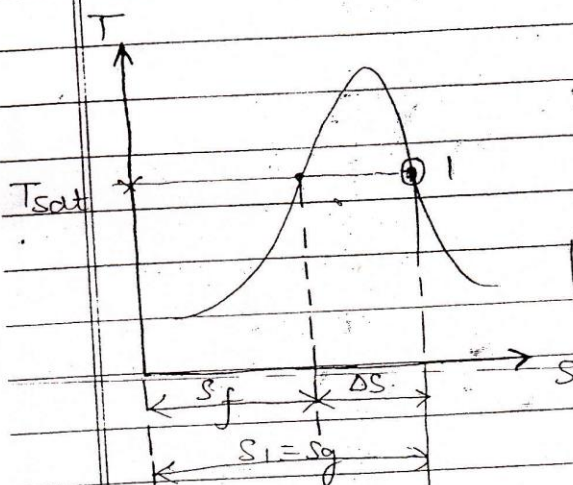


✓ Superheated vapour can be treated as an ideal gas with less error. Hence

$$h_1 = h_g + (C_p)_{\text{vapour}} (T_1 - T_{\text{sat}})$$

[ENTROPY AT VARIOUS POINTS]

E① when the point is on saturated vapour curve.



$S_1 = S_g + \Delta S$
 ΔS = entropy change during phase transformation

phase transformation can be treated as a reversible process

We know for reversible process

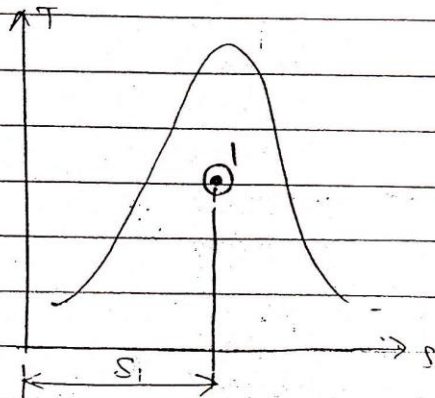
$$dS = \frac{dQ}{T}$$

$$\Delta S = \frac{L.H}{T_{\text{sat}}}$$

$$S_1 = S_g + \frac{L.H}{T_{\text{sat}}}$$

$$\checkmark S_1 = S_g = S_f + \frac{L.H}{T_{\text{sat}}}$$

CASE (2) $\rightarrow$ when the point is in wet region. ²⁸¹ 78



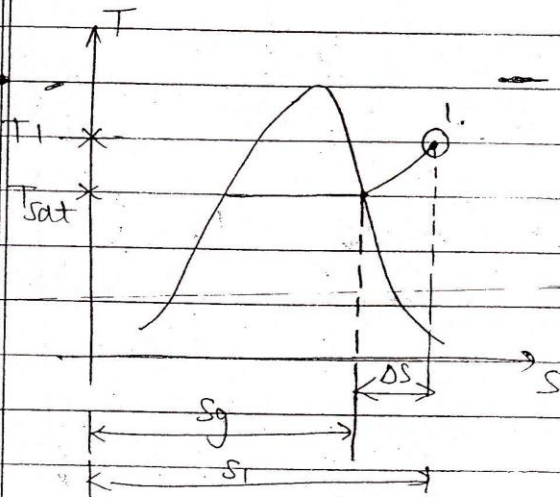
$$S_1 = S_f + x (S_g - S_f)$$

$$S_1 = S_f + x (L.H)_{T_{sat}}$$

Gnp

CASE (3) $\rightarrow$

when the point is in superheated region.



$$\Delta S = C_p \ln \left(\frac{T_1}{T_i} \right) - R \ln \left(\frac{P_f}{P_i} \right)$$

$$\Delta S = (C_p)_{vap} \ln \left(\frac{T_1}{T_{sat}} \right) - R \ln(1)$$

$$\Delta S = (C_p)_{vap} \ln \left(\frac{T_1}{T_{sat}} \right)$$

$$S_1 = S_g + \Delta S$$

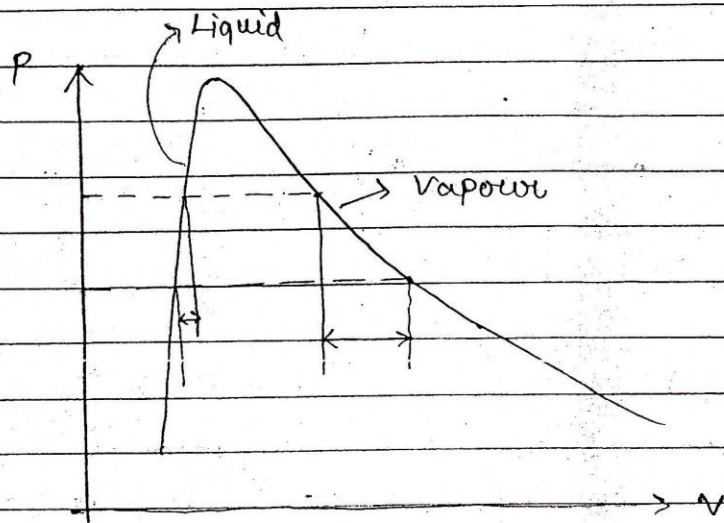
$$S_1 = S_g + (C_p)_{vap} \ln \left(\frac{T_1}{T_{sat}} \right)$$

Saturday (10-7) - Capt Hing

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[P-V diagram for a pure substance]



✓[MOLLIER DIAGRAM]

↓

[h-s diagram for a pure substance]

* h-s diagram for a pure substance is known as Mollier diagram

we know

$$Tds = dh - PvdP$$

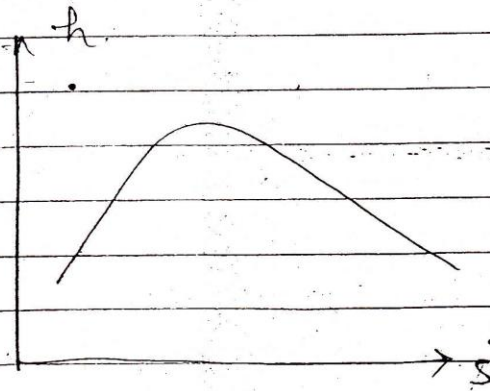
if $P = \text{constant}$
 $dP = 0$

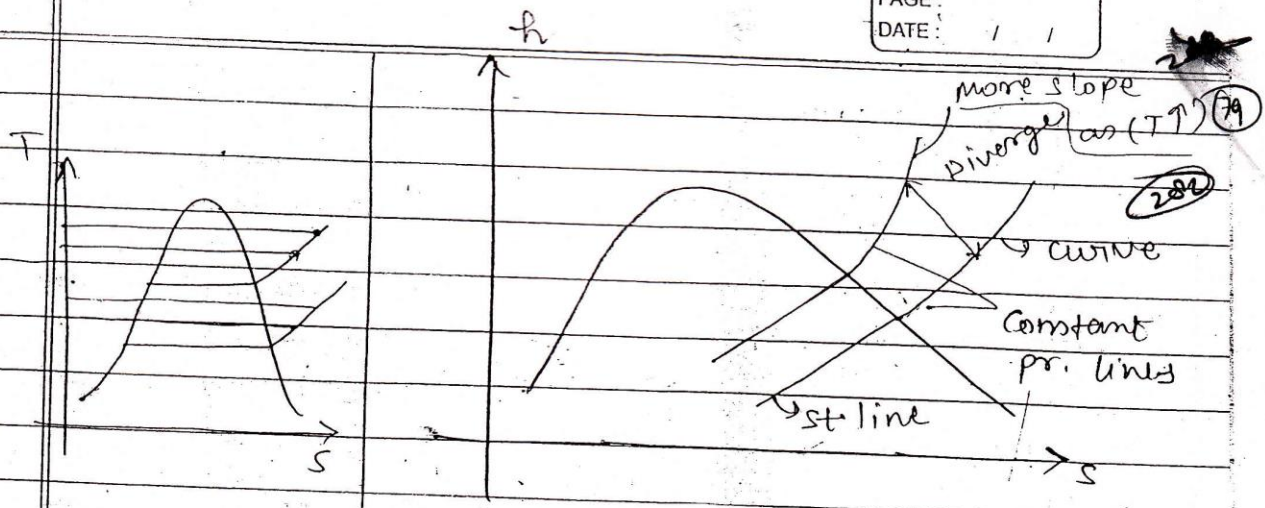
$$Tds = dh$$

$$\boxed{\frac{dh}{ds} = T}$$

↓

Slope of constant pressure line = Temp (T).



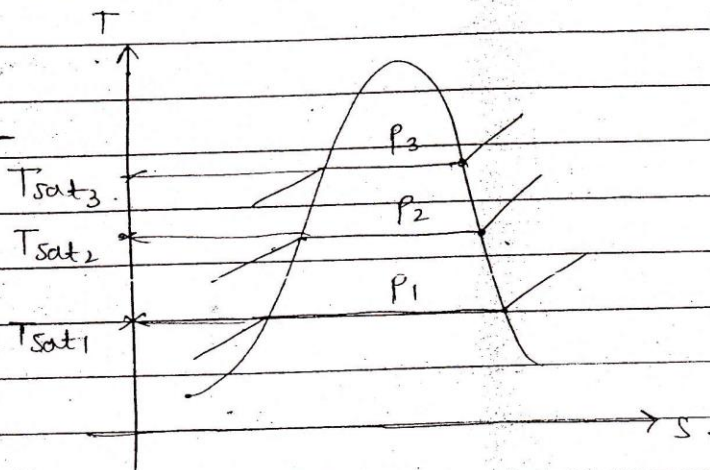


(General substances)	(water)
Solid-Liquid $\rightarrow$ expands	ice (s) $\rightarrow$ water (L)
Liquid-Solid $\rightarrow$ contracts	$\rightarrow$ contracts
	water (L) $\rightarrow$ ice (s)
	$\rightarrow$ expands

Solid $\rightarrow$ Liquid (Melting)
 Liquid $\rightarrow$ Solid (Freezing)
 Liquid $\rightarrow$ Vapour (Vaporization)
 Vapour $\rightarrow$ Liquid (Condensation)
 Solid $\rightarrow$ Vapour (Sublimation)
 Vapour $\rightarrow$ Solid (Ablimation)

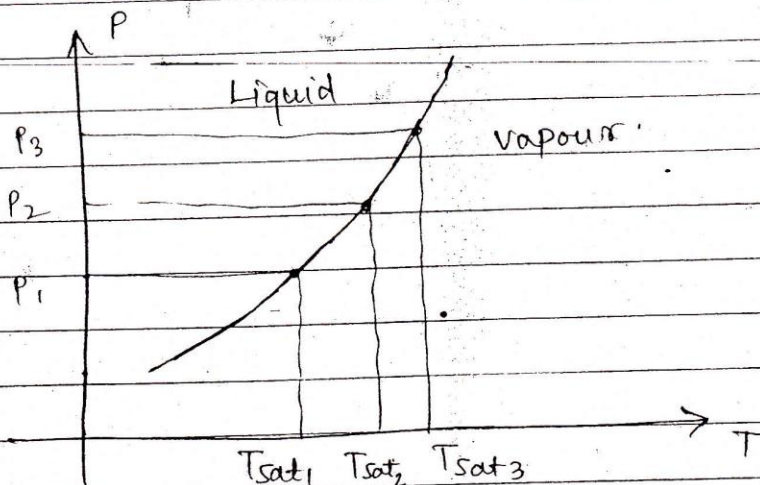
* [P-T diagram for a pure substance] *

Let's take T-s diagram first -



$P(T_{sat1}, P_1), (T_{sat2}, P_2), (T_{sat3}, P_3)$

Let's plot these points on P-T diagram



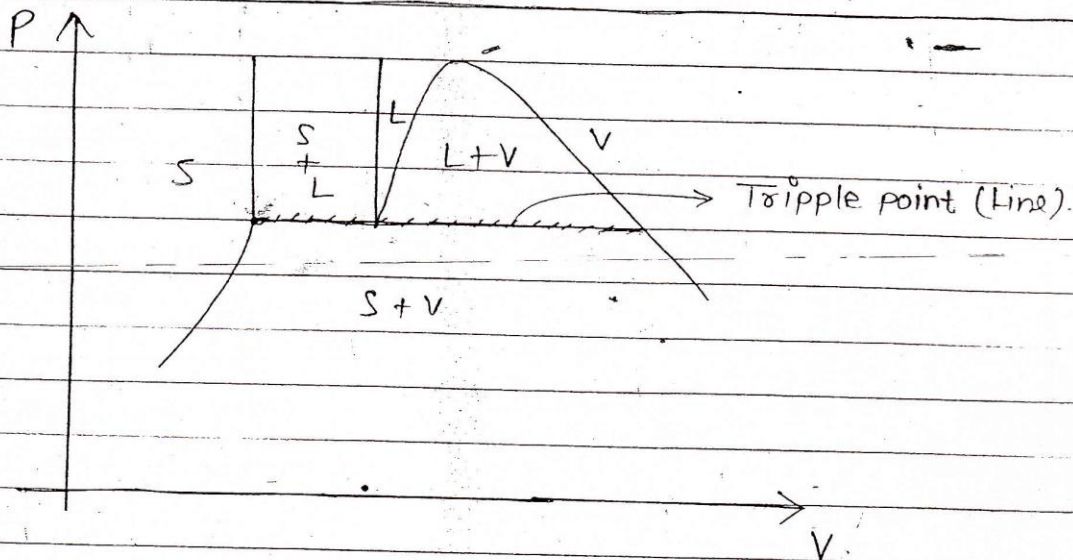
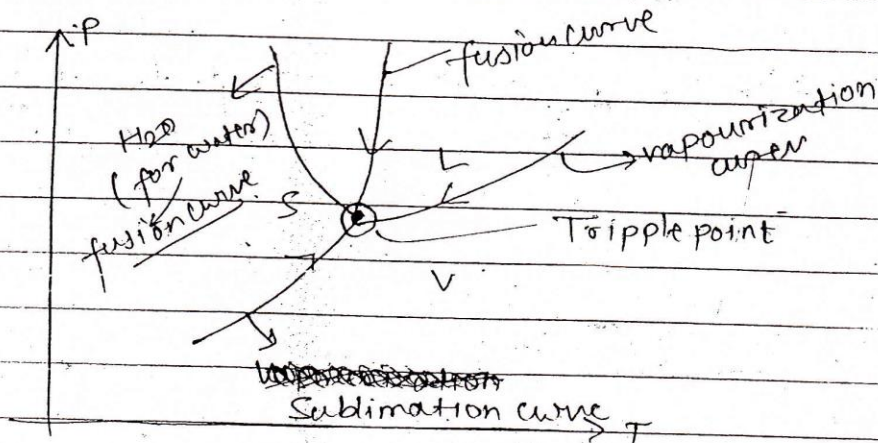
The above curve is for Liquid & vapour

If we plot a curve for liquid & solid
and for solid and vapour we get

almost similar curves on P-T diagram.

203
(80)

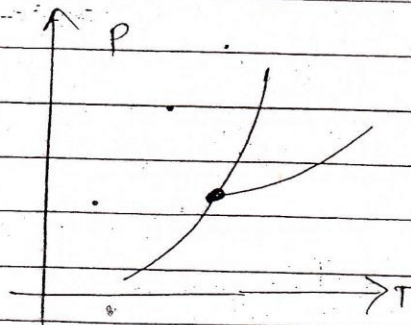
The three curves meet each other at a common point on ~~the~~ P-T diagram. This point is known as TRIPPLE POINT.



According to Gibbs's phase rule the degree of freedom at tripple point is zero, i.e.

No intensive property

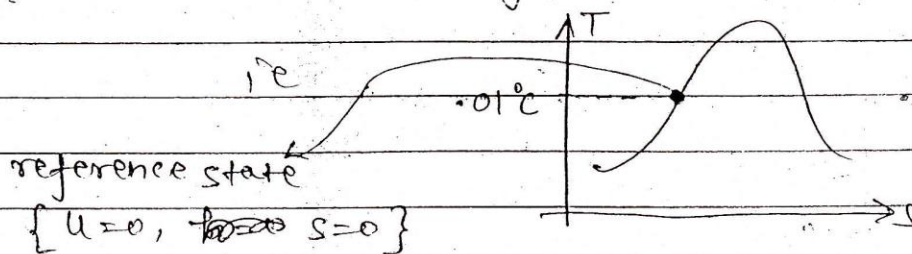
can be changed at tripple point, i.e. at tripple point all intensive properties are fixed.



NOTE $\rightarrow$ Tripple point is a point on P-T diagram and it is a line on P-V diagrams

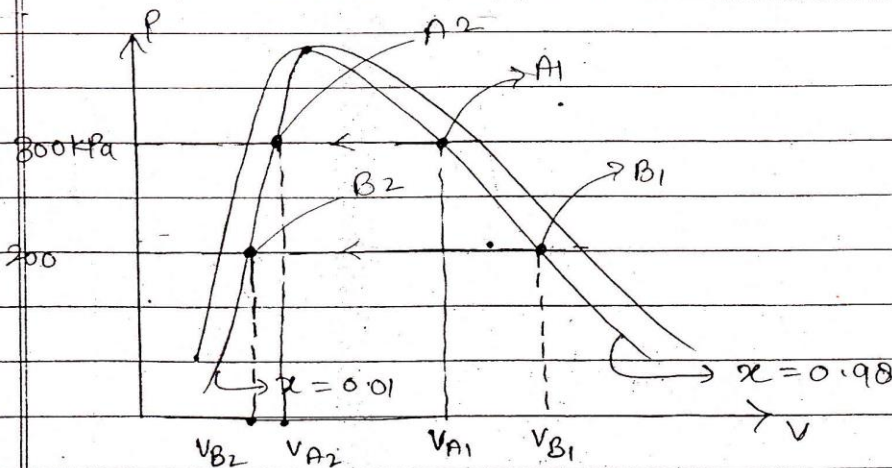
* [Reference state in standard steam tables] *

The internal energy and entropy of saturated liquid at tripple point temp (0.01°C) is arbitrarily taken as zero.



[WORKBOOK] Ch-6

Q.1



$$v_{A1} < v_{B1}, \quad v_{A2} > v_{B2}$$

Q.2

Given $v_v = 8 v_l$

$$v_f = v_l = 0.001044$$

$$v_g = v_v = 1.6729$$

Eduwrte $x = ?$

204
81

we know $\alpha = \frac{m_v}{m_l + m_v} = \frac{m_v}{m_v (1 + \frac{m_l}{m_v})}$

$$\alpha = \frac{1}{1 + \frac{m_l}{m_v}} \quad \text{--- (1)}$$

we know $\frac{V}{m} = v$ OR $m = \frac{V}{v}$

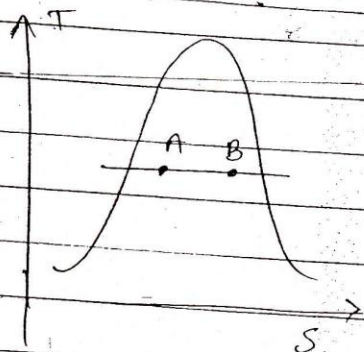
$m_l = \frac{V_l}{v_l}$ & $m_v = \frac{V_v}{v_v}$ Putting in (1)

$$\alpha = \frac{1}{1 + \frac{V_l}{v_l} \times \frac{v_v}{V_v}}$$

$$\alpha = \frac{1}{1 + \frac{1}{8} \times \frac{1.6729}{0.001044}}$$

$$\alpha = 0.00496 \approx 0.005$$

③



$u_A = u_B$ (given)

$m_A \cdot u_A = m_B \cdot u_B$

$$\frac{m_A}{m_B} = \frac{u_B}{u_A}$$

$$\frac{m_A}{m_B} = \frac{u_f + \alpha_A (u_g - u_f)}{u_f + \alpha_B (u_g - u_f)}$$

if $\frac{m_A}{m_B}$ has to be maximum -

$\alpha_A \rightarrow \text{maximum} = 1$

$\alpha_B \rightarrow \text{minimum} = 0$

$$\begin{aligned} \frac{m_A}{m_B} &= \frac{u_f + 1 (u_g - u_f)}{u_f + 0 (u_g - u_f)} = \frac{u_f + u_g - u_f}{u_f} \\ &= \frac{u_g}{u_f} = 3.99 \end{aligned}$$

PS. U

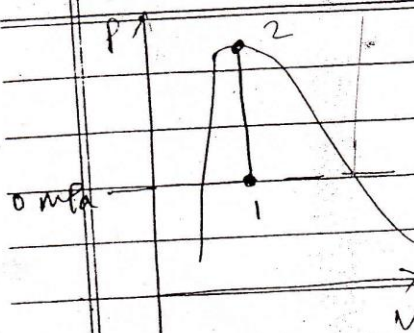
$$P_{cr.} = 221.2 \text{ bar}$$

$$T_{cr.} = 374.1^\circ\text{C}$$

> at critical point

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given $u_1 = u_2$

and $u_2 = 0.00317$

So $u_1 = u_2 = 0.00317$

$$u_1 = u_f + x(u_g - u_f)$$

$$0.00317 = 0.001452 + x(0.01802 - 0.001452)$$

$$x = 10.3\% \text{ Ans}$$

2) Given $\rightarrow \rho = 36.12 \text{ kg/m}^3$ at -30°C

$$\rho_f = 1377 \text{ kg/m}^3$$

$$\rho_g = 7.379$$

we know $u = u_f + x(u_g - u_f)$

$$\frac{1}{\rho} = \frac{1}{\rho_f} + x\left(\frac{1}{\rho_g} - \frac{1}{\rho_f}\right)$$

$$\frac{1}{36.12} = \frac{1}{1377} + x\left(\frac{1}{7.379} - \frac{1}{1377}\right)$$

$$x = 0.2$$

$x = 0.4$
$V_1 = 1 \text{ m}^3$

initial

$h_2 = 125.77$
kJ/kg

final

$$h_1 = h_f + x(h_g - h_f)$$

$$h_1 = 504.7 + 0.4(2706.7 - 504.7)$$

$$h_1 = 1385.5 \text{ kJ/kg}$$

$$Q_p = \Delta h = h_2 - h_1$$

$$Q_p = 125.77 - 1385.5$$

$$Q_p = -1259.7 \text{ kJ/kg}$$

$$v = \frac{V}{m} \Rightarrow m = \frac{V}{v}$$

$$\text{but } v = v_f + x(v_g - v_f)$$

$$x = \frac{v - v_f}{v_g - v_f} = 0.355$$

$$m = \frac{V}{v} = \frac{1}{0.355} = 2.82 \text{ kg}$$

$$\text{Total heat transfer} = 2.82 (-1259.7) \text{ kJ} \\ = -3550 \text{ kJ}$$

⑨ $S_f = 1.3026 \frac{\text{kJ}}{\text{kg-K}}$ at $P_1 = 1 \text{ bar}$
 $T_1 = 99.63^\circ\text{C}$

$$S_g = ?$$

$$\text{we know } S_g - S_f = \frac{L.H}{T}$$

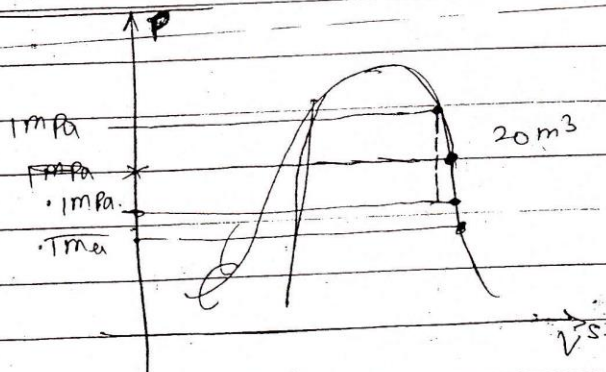
$$L.H = h_g - h_f = 2675.5 - 417.5$$

$$S_g - S_f = 6.059$$

$$S_g = 6.059 + 1.3026$$

10

$$Q_v = \Delta U$$



⑫ $h_f = 762.0 \frac{\text{kJ}}{\text{kg}}$

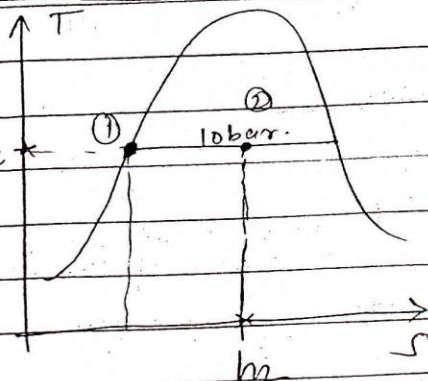
$$h_g = 2778.1 \frac{\text{kJ}}{\text{kg}}$$

$$Q_p = 10 \text{ kJ}$$

$$P = \frac{10}{2} \frac{\text{kJ}}{\text{kg}} = 5 \frac{\text{kJ}}{\text{kg}}$$

$$Q_p = h_2 - h_1$$

$$h_2 = h_f + x(h_g - h_f)$$



$$Q_p = h_2 - h_1$$

$$h_2 = Q_p + h_1$$

$$h_2 = 5 + 762.0 = 767.0$$

$$h_2 = h_f + x(h_g - h_f)$$

$$767.0 = 762.0 + x(2778.1 - 762.0)$$

$$x = 0.00248$$

$$x = \frac{m_v}{m_v + m_l} \Rightarrow m_v = 0.00248 \times 1002$$

$$m_v = 4.96 \times 10^{-3} \text{ kg}$$

$$m_v = 4.96 \text{ gm}$$

14

18

$$m = 10$$

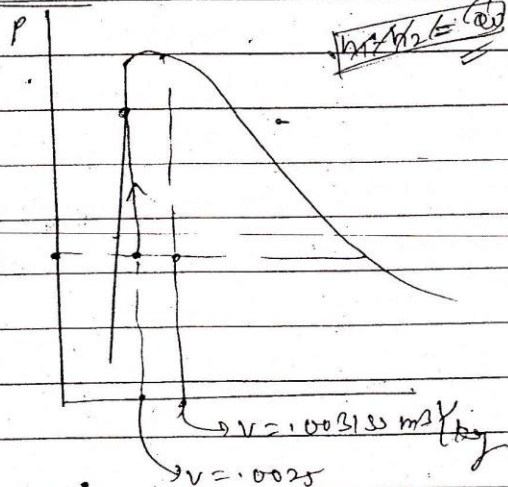
$$V = 0.025$$

$$P = 0.1$$

$$V_{act} = \frac{V}{m}$$

$$V_{act} = \frac{0.025}{10}$$

$$= 0.0025$$



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for min. x

process must be

isentropic

$$S_1 = S_2$$

$$S_1 = 6.9362 \text{ kJ/K-K}$$

$$S_2 = S_f + x(S_g - S_f)$$

$$= 1.091 + x(7.0939 - 1.091)$$

$$x = 0.898$$

(25)

$$h = u + pv$$

for liquids (f) $\rightarrow u_f \rightarrow \text{small}$

$$h_f = u_f + (p_f v_f) \rightarrow \text{negligible}$$

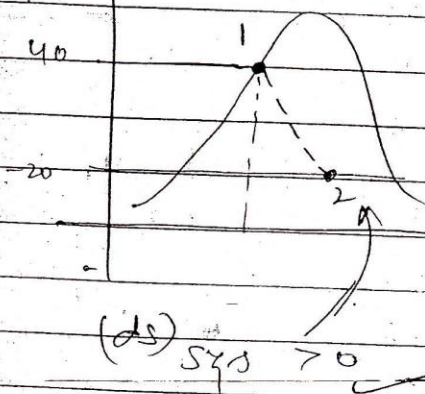
$$h_f \approx u_f \quad (h_f \text{ slightly more than } u_f)$$

5th is ~~h_f~~

3rd is u_f

we know $u_g > u_f$ & $h_g > h_f$

Q.26 $\rightarrow$ $\uparrow T$



Throttling is irreversible

$$\Rightarrow (ds)_{\text{unrev}} > 0$$

$$\Rightarrow (ds)_{\text{sys}} + (ds)_{\text{sur}} > 0$$

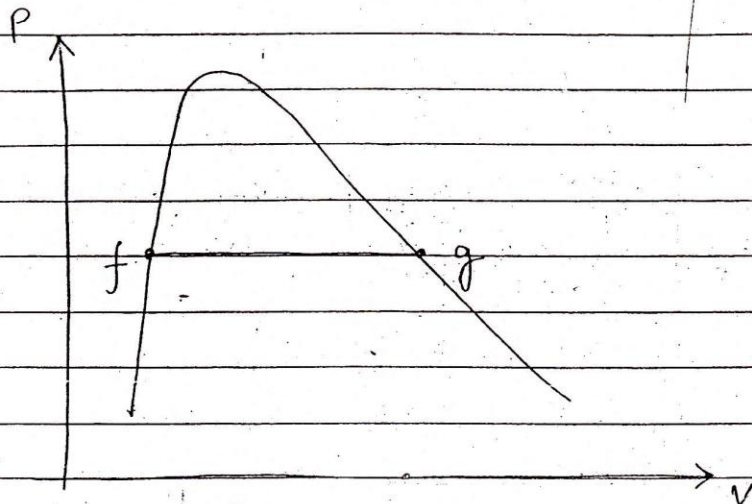
$$h_1 = 371.43$$

$$h_2 = h_f + x(h_g - h_f)$$

$$h_2 = 89.05 + x(1418 - 89.05) = h_1$$

$$x = 0.212$$

[CLAUSIUS - CLAPEYRON EQUATION]



we know

$$\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T \rightarrow \text{Maxwell's eqn}$$

* During phase change pressure & temp are independent of volume (from P-v diagram)

$$\frac{dP}{dT} = \frac{S_g - S_f}{V_g - V_f} = \frac{(L.H)/T}{(V_g - V_f)}$$

$$\boxed{\frac{dP}{dT} = \frac{L.H}{T(V_g - V_f)}} \quad \text{--- Clapeyron eqn ---}$$

* we know $V_f \ll V_g$

$$\Rightarrow \frac{dP}{dT} \approx \frac{L.H}{T(V_g)} \quad \text{--- vapour ---}$$

for vapour (treating as ideal gas)

$$PV = nRT$$

$$\frac{V}{n} = \frac{RT}{P} \Rightarrow v = \frac{RT}{P}$$

$$\Rightarrow \frac{dp}{dT} = \frac{L.H.}{T \left(\frac{RT}{P} \right)}$$

(84)

$$\frac{dp}{dT} = \frac{P(L.H.)}{RT^2} \rightarrow \text{Clausius-Clapeyron's equation}$$

expected

JOULE - THOMPSON COEFFICIENT: (H)

$$H = \left(\frac{\partial T}{\partial P} \right)_H$$

* we know

$$Tds = dH - vdp$$

$$dH = Tds + vdp \quad \text{--- (1)}$$

from 2nd Tds eqn

$$Tds = C_p dT - T \left(\frac{\partial v}{\partial T} \right)_P dP$$

putting in (1)

$$dH = C_p dT - T \left(\frac{\partial v}{\partial T} \right)_P dP + vdp$$

$$dH = C_p dT - \left[T \left(\frac{\partial v}{\partial T} \right)_P - v \right] dP \quad \text{--- (2)}$$

for an ideal gas.

$$PV = mRT$$

$$\left(\frac{\partial v}{\partial T} \right)_P = \frac{mR}{P}$$

$$\text{but } \frac{mR}{P} = \frac{v}{T}$$

$$\Rightarrow \left(\frac{\partial v}{\partial T} \right)_P = \frac{v}{T} \Rightarrow$$

$$T \left(\frac{\partial v}{\partial T} \right)_P - v = 0 \quad \text{putting in (2)}$$

$$dH = C_p dT - 0$$

$$dH = C_p dT \rightarrow \text{for an ideal gas.}$$

we know

$$dh = C_p dT - \left[T \left(\frac{\partial v}{\partial T} \right)_P - v \right] dp$$

considering $h \rightarrow \text{constant}$ [Throttling process]

$$0 = C_p dT - \left[T \left(\frac{\partial v}{\partial T} \right)_P - v \right] dp$$

$$C_p dT = \left[T \left(\frac{\partial v}{\partial T} \right)_P - v \right] dp$$

$$\left(\frac{\partial T}{\partial P} \right)_h = \frac{1}{C_p} \left[T \left(\frac{\partial v}{\partial T} \right)_P - v \right]$$

↓

$$\boxed{\mu = \frac{1}{C_p} \left[T \left(\frac{\partial v}{\partial T} \right)_P - v \right]}$$

expected.

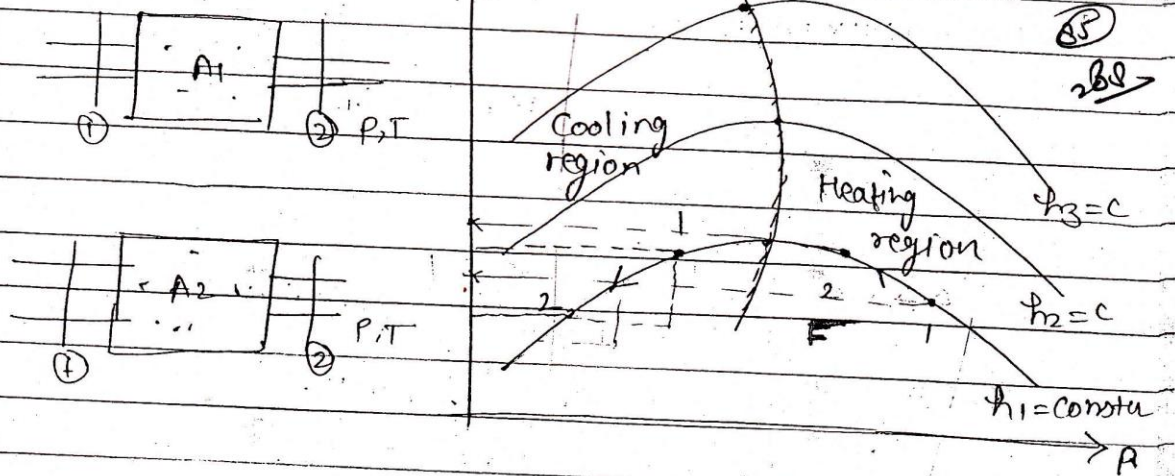
for an ideal gas.

$$\left[T \left(\frac{\partial v}{\partial T} \right)_P - v \right] = 0 \quad \text{[Proved earlier]}$$

$$\Rightarrow (\mu)_{\text{ideal gas}} = \frac{1}{C_p} (0)$$

$$\boxed{(\mu)_{\text{ideal gas}} = 0}$$

**



for cooling region

$$\left(\frac{\partial T}{\partial P} \right)_h = \text{slope +ve.}$$

so $\mu_{\text{cooling region}} = \left(\frac{\partial T}{\partial P} \right)_h = +ve$

for heating region

$$\left(\frac{\partial T}{\partial P} \right)_h = \text{slope -ve}$$

$$(\mu)_{H.R} = \left(\frac{\partial T}{\partial P} \right)_h = -ve$$

*

In cooling region μ is +ve and in heating region μ is -ve.

* for ideal gas $\mu = 0$.

[MIXTURE OF IDEAL GASES]

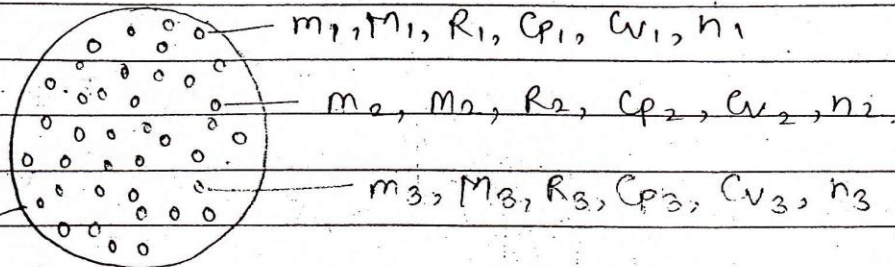


fig: Mix of gases

$$\begin{aligned}
 &V_1 = V_2 = V_3 = V \\
 &P = P_1 + P_2 + P_3 + \dots
 \end{aligned}$$

Mole fraction $\rightarrow$ It is the ratio of no. of moles of a gas to the total no. of moles.

$$x = \frac{n}{\sum n} \Rightarrow x = \frac{n_1}{n_1 + n_2 + n_3 + \dots}$$

$\rightarrow$ As the system is in equilibrium

$$T_1 = T_2 = T_3 = T$$

& $V_1 = V_2 = V_3 = V \rightarrow$ Obviously for a gas mix.

$P = P_1 + P_2 + P_3 \dots$ (Dalton's law of partial pressure)

V, P, T
 $\sum m, M_e$
 $\sum n, R_e$
 $\rightarrow$ equivalent system [Supposed to have single gas]

we know $PV = n \bar{R} T$

for ① $P_1 V = n_1 \bar{R} T \quad \text{--- ①}$

for equivalent sys $PV = \sum n \bar{R} T \quad \text{--- ②}$

Eduwrtte :

$$\frac{eq^n (1)}{eq^n (2)} \Rightarrow \frac{P_1 V}{P V} = \frac{n_1 \bar{R} T}{\sum n \bar{R} T}$$

(86)

89

$$\frac{P_1}{P} = \frac{n_1}{\sum n} = x_1$$

$$\frac{P_1}{P} = x_1 \Rightarrow \begin{array}{|l} P_1 = P x_1 \\ P_2 = P x_2 \\ P_3 = P x_3 \end{array} \begin{array}{l} * \\ \checkmark \\ \checkmark \end{array}$$

EQUIVALENT GAS CONSTANT (R_e)

for I gas

$$P_1 V = m_1 R_1 T$$

II gas

$$P_2 V = m_2 R_2 T$$

III

$$P_3 V = m_3 R_3 T$$

$$\begin{array}{c} + \quad - \quad + \end{array}$$

$$(P_1 + P_2 + P_3 + \dots) V = (m_1 R_1 + m_2 R_2 + m_3 R_3 + \dots) T$$

$$P V = (m_1 R_1 + m_2 R_2 + \dots) T \quad \text{--- (1)}$$

for equivalent system.

$$P V = \sum m R_e T \quad \text{--- (2)}$$

Comparing (1) & (2)

Vimp

$$\sum m \cdot R_e = (m_1 R_1 + m_2 R_2 + m_3 R_3 + \dots)$$

* Equivalent molecular weight (Me)

$$\text{for I gas} \quad m_1 = n_1 M_1$$

$$\text{II " } \quad m_2 = n_2 M_2$$

$$\text{III " } \quad m_3 = n_3 M_3$$

$$\text{Adding} \rightarrow (m_1 + m_2 + m_3 + \dots) = n_1 M_1 + n_2 M_2 + \dots$$

$$\text{or } \Sigma m = n_1 M_1 + n_2 M_2 + \dots \quad \text{--- (1)}$$

for equivalent system.

$$\Sigma m = \Sigma n \cdot Me \quad \text{--- (2)}$$

Comparing (1) & (2)

$$\Sigma n \cdot Me = n_1 M_1 + n_2 M_2 + n_3 M_3 + \dots$$

$$Me = \frac{n_1 M_1 + n_2 M_2 + \dots}{\Sigma n}$$

$$Me = \left(\frac{n_1}{\Sigma n} \right) M_1 + \left(\frac{n_2}{\Sigma n} \right) M_2 + \dots$$

$$\checkmark \boxed{Me = x_1 M_1 + x_2 M_2 + x_3 M_3 + \dots}$$

* ENTROPY CHANGE DUE TO DIFFUSION

Gas A	Gas B
P	P
T	T

Diffusion $\Rightarrow$

(87)

290

$$\Delta S = -n_1 \bar{R} \ln \alpha_1 - n_2 \bar{R} \ln \alpha_2 + \dots$$

where

$$x_1 = \frac{n_1}{n_1 + n_2 + \dots}$$

[WORKBOOK] {CH-7}

①

$$\frac{dP}{dT} = 17.69$$

$$T = 293 \text{ K}, \quad v_g = 0.0008157$$

$$v_f = 0.0358$$

$$L.H = h_g - h_f = ?$$

Clapeyron's eqn

$$\frac{dP}{dT} = \frac{L.H}{T(v_g - v_f)}$$

$$17.69 = \frac{L.H}{293(-)}$$

$$L.H = 181.5 \text{ KJ/kg}$$

②

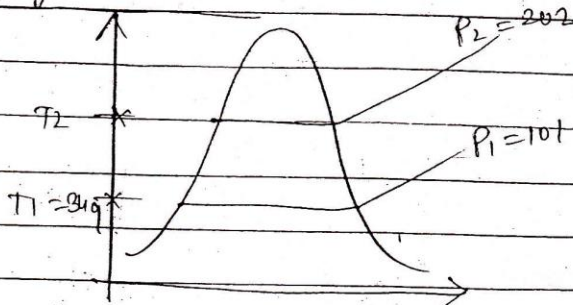
$$L.H = 195$$

$$R = 0.055$$

we know,

$$\frac{dP}{dT} = \frac{P(L.H)}{RT^2}$$

$$\frac{dP}{P} = \frac{L.H}{R} \cdot \frac{dT}{T^2}$$



assume L.H. to be constant (although it is not)

$$\int_{P_1}^{P_2} \frac{dP}{P} = \frac{L.H.}{R} \int_{T_1}^{T_2} \frac{dT}{T^2}$$

$$\ln \frac{P_2}{P_1} = \frac{L.H.}{R} \left(-\frac{1}{T} \right)_{T_1}^{T_2}$$

$$\ln \left(\frac{P_2}{P_1} \right) = -\frac{L.H.}{R} \left(\frac{1}{T_2} - \frac{1}{T_1} \right)$$

$$\ln \left(\frac{202}{10.1} \right) = \frac{-195}{0.055} \left(\frac{1}{T_2} - \frac{1}{349} \right)$$

$$T_2 = 574.54 \text{ K} \quad \text{Ans (d)}$$

Q.4 → $G = H - TS$

$$F = U - TS$$

$$G - F = H - TS - U + TS$$

$$G - F = H - U$$

$$G - F = (H) - (U)$$

$$= (U + PV) - U$$

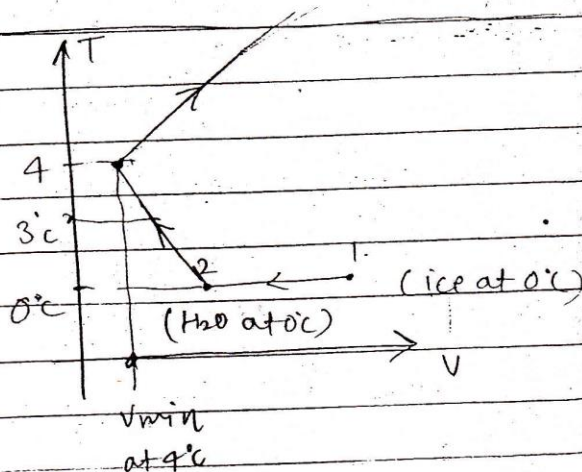
$$G - F = PV$$

for ideal gas $G - F = nRT$ (b) Ans

62) $\left(\frac{\partial S}{\partial P} \right)_T = - \left(\frac{\partial V}{\partial T} \right)_P$

↓
from Maxwell's
eqn

we see that
at 3°C → slope is
negative



$$\left(\frac{\partial S}{\partial P}\right)_T = - \left(-\frac{\partial V}{\partial T}\right)_P > 0 \text{ at } 3^\circ\text{C}$$

(88)

(291)

after 4°C slope is +ve

$$\left(\frac{\partial S}{\partial P}\right)_T = - \left(\frac{\partial V}{\partial T}\right)_P < 0 \text{ at } 5^\circ\text{C}$$

Ans (d)

$$Q.7 \rightarrow \ln P = A - \frac{B}{T}$$

Diffo

$$\frac{1}{P} \frac{dP}{dT} = 0 = B \left(-\frac{1}{T^2}\right)$$

$$\frac{dP}{dT} = \frac{PB}{T^2} \quad \text{--- (1)}$$

$$\text{but } \frac{dP}{dT} = \frac{LH}{T(v_g - v_f)} \quad \text{--- (2)}$$

$$\text{from (1) and (2) } \left\{ LH = \frac{P_{\text{sat}} \cdot B \cdot v_{fg}}{T} \right\} \text{ Ans (b)}$$

$$(9) \left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V \text{ --- Maxwell}$$

$$PV = nRT$$

$$PV = RT$$

$$\left(\frac{\partial P}{\partial T}\right)_V = \frac{R}{V}$$

$$(10) \Delta S = -n_1 \bar{R} \ln x_1 - n_2 \bar{R} \ln x_2$$

$$= -\bar{R} [n_1 \ln x_1 + n_2 \ln x_2]$$

$$n_1 = 1, n_2 = 4 \Rightarrow x_1 = 0.2 \text{ \& } x_2 = 0.8$$

$$\Delta S = -8.314 [1 \ln(0.2) + 4 \ln(0.8)]$$

$$\Delta S = 20.8 \text{ Ans (b)}$$

Eduwrte

12-(c)

15 (b)

3-(d)

16. (a)

4 (c)

PAGE :

DATE : / /

$$(17) \quad c_p - c_v = \frac{TV\beta^2}{k_T}$$

$$c_p - c_v = \frac{(273+25)(0.000114)(5 \times 10^{-5})^2}{0.6 \times 10^{-12}}$$

$$c_p - c_v = 9.8$$

$$\frac{c_p}{c_v} = 1.024$$

$$c_p = 9.8 + c_v$$

$$c_v = \frac{c_p}{1.024}$$

$$c_p = \frac{c_p}{1.024} = 9.8$$

$$\{c_p = 421 \text{ J/kg-K}\}$$

Q-18 → 1- (D)

(19)

(a)

The end

